Old Business. Two examples of the equation of a line were given in the first lecture, but not on slides. They are included here so that they will automatically be made available on the web. The first example is the relation between F degrees Fahrenheit and C degrees Celsius:

$$F = \frac{9}{5}C + 32. \tag{1}$$

This is written in the **slope-intercept** form that *looks* simpler because it doesn't need parentheses. It is clear that $C = 0$ gives $F = 32$ — this is the **intercept**. The **slope**, defined as the ratio of the difference of two values of F to the difference of the corresponding values of C, is $9/5$. Equation (1) is usually derived by simplifying the **two-point** form

$$\frac{F - 32}{C - 0} = \frac{212 - 32}{100 - 0} \tag{2}$$

obtained from the freezing point of water

$$(C, F) = (0, 32)$$

and the boiling point of water

$$(C, F) = (100, 212).$$

Once the slope is known, a **point-slope** form is obtained for each point on the line. For example, the point $(C, F) = (-40, -40)$ gives

$$F + 40 = \frac{9}{5}(C + 40), \tag{3}$$

and $(C, F) = (10, 50)$, which is best suited for working with temperatures in the temperate zone, gives

$$F - 50 = \frac{9}{5}(C - 10). \tag{4}$$

To see that these define the same line, you need only verify that (2), (3) and (4) all simplify to (1).

The other example was the special **intercept** form of a line that doesn't pass through the origin and isn't parallel to either axis:

$$\frac{x}{a} + \frac{y}{b} = 1. \tag{5}$$

It is *visually evident* that (5) is of the first degree in both x and y, and that the points

$$(x, y) = (a, 0) \text{ and } (x, y) = (0, b) \tag{6}$$

both satisfy (5). Thus, (5) is the equation of a line through the points (6), but there is only one such line, so any equation is equivalent to (5).

Functions. The theoretical basis of Calculus is the idea of **function**. There are variations in the definition that turn out to be minor although they may give different answers when testing whether a particular example is a function. We are more concerned with **using** the **idea** behind the definition than arguing about technicalities. Mathematicians try to use definitions that are both precise and useful, but sometimes things **look** different when they are used. This is called **abuse of notation**, which sounds awful but it protects you from being abused by notation.

Here is the definition in our textbook:

A function is a rule that assigns to each element in a set A **one and only one** (emphasis added) element in a set B.

The set A is called the **domain** of the function.

No name is given to the set B, but the subset of elements of B that are assigned to something in A is

called the **range** of the function.

Example Let A and B each be the set of all real numbers and let the function assign to $x \in A$ the number x^2. This is a function because we **know** that we can find x^2 whenever we are given a number x. Although all numbers were allowed in B, we also know that, given y, there is an x with $y = x^2$ **if and only if** (a phrase used so frequently that it is shortened to **iff**) $y \geq 0$. Thus, the range of this function is the interval $[0, \infty)$.

Once functions are accepted as objects, we feel that they should be allowed to have names. The name f is often used when we can't think of anything better and the element of B assigned to $x \in A$ is called $f(x)$.

In this course, most functions will have an **implicit domain** in the set of all real numbers. If a function f is defined by writing an expression for $f(x)$, only values for which the formula can be evaluated are allowed in the domain. It is a common form of abuse of notation to ask for the (largest) domain of an expression.

In applications, there may be other restrictions. Example 3 in Section 2.1 deals with making a box from a cardboard rectangle. The construction of the box is described as: (1) cutting out a square of size x at each corner; (2) folding along certain edges in the resulting figure. Here, we need $x > 0$ in order to realize (1), and x less than half the shortest side of the original rectangle in order to identify the lines needed in (2). In this example, both the formula for the value of the function giving the volume of the box and the domain of this function are found as part of the process that turns a physical description of a construction into mathematics.

The set of points whose coordinates are $(x, f(x))$ for some x is called the **graph** of f. Graphs of functions are examples of **curves** in the plane, but a curve can only be the graph of a function (on some domain) if vertical lines meet the curve in at most one point.

Algebra of functions. If f and g are two functions, then for every x in the intersection of the domain of f and the domain of g, $f(x)$ and $g(x)$ are **numbers**. Forming the sum of these two gives a new rule for associating a number with x. The function defined in

this way is called the sum of the functions f and g. In symbols, $f+g$ **is defined by**

$$(f+g)(x) = f(x) + g(x).$$

One may define the difference $f-g$, the product fg, and the quotient f/g similarly. For the quotient, the domain has the additional condition that $g(x) \neq 0$.

Composition of functions. The **black box** description of figure 2.1 of the text, or the **arrow** description of figure 2.2, or the notation $A \xrightarrow{g} B$ suggests a **composition** $f \circ g$ visualized as

$$A \xrightarrow{g} B \xrightarrow{f} C.$$

A short definition on elements is

$$(f \circ g)(x) = f\big(g(x)\big).$$

The natural domain of $f \circ g$ consists of those x in the domain of g such that $g(x)$ is in the domain of A. For many examples, this can be simplified.

Analysis. The formulas of the differential calculus will be given by specifying how the **derivative** (to be defined in section 2.6) of algebraic combinations like fg or compositions like $f \circ g$ is related to the f, g and their derivatives. In order to **use** these rules, it is necessary to see these patterns when a function h is described by giving a **formula** for $h(x)$. This makes exercises 35 – 42 of Section 2.2 extremely important. The warning that "the answer is not unique" hints at the **magic** of Calculus: different ways of describing a function in order to find its derivative will always lead to the same derivative. This works because there is a definition of the derivative that fixes the meaning independent of the formal rules used in a calculation.