Section 12.3

The exercises from sections 12.1 and 12.2 will not be done here, but should be done in recitation. If there are any aspects of the basic properties of the trigonometric functions that you need to review, it will help to **discuss** the exercises. However, presentation here will do little more than show the answer. Moving on to the calculus seems a better use of lecture time.

Typically, a function $f(x)$ will be given and $f'(x)$ is to be found.

Exercise 2

$$f(x) = \sin 5x$$
$$f'(x) = \big(\cos 5x\big)\big(5\big) = 5\cos 5x$$

Exercise 6

$$f(x) = \cos \pi x^2$$
$$f'(x) = \big(-\sin \pi x^2\big)\big(2\pi x\big) = -2\pi x \sin \pi x^2$$

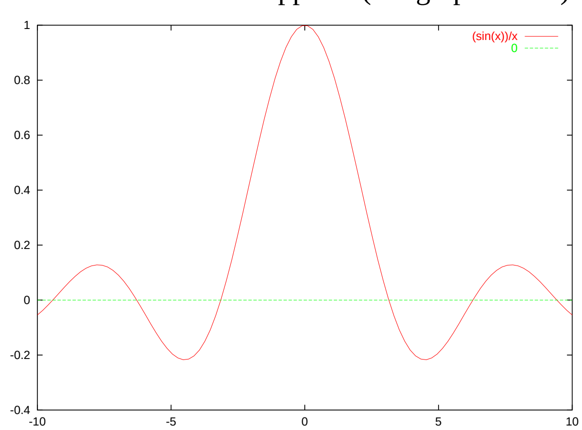
Exercise 17

$$f(x) = e^x \sec x$$
$$f'(x) = (e^x)(\sec x \tan x) + (\sec x)(e^x)$$
$$f'(x) = e^x \cdot (\sec x) \cdot (\tan x + 1)$$

Exercise 25

$$f(x) = \frac{\sin x}{x}$$
$$f'(x) = \frac{(x)(\cos x) - (\sin x)(1)}{x^2}$$
$$f'(x) = \frac{x\cos x - \sin x}{x^2}$$

This formula is not defined when $x = 0$, but the limit as $x \to 0$ exists and is equal to 1 (because **radian** measure of angles is **always used** for trigonometric functions in calculus). This use of differentiation formulas is also only valid for $x \neq 0$. More advanced methods are required to evaluate the derivative at $x = 0$ and to show that the curve is as smooth as it appears (see graph below).



Exercise 31 This asks for the tangent line to the graph of $y = \cot 2x$ at the point $(\pi/4, 0)$. First, check that the information has been correctly transcribed and interpreted by verifying that the point lies on the curve. If $x = \pi/4$, the $2x = \pi/2$, and

$$\cot\frac{\pi}{2} = \frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}} = 0,$$

since $\cos u = 0$ and $\sin u = 1$ at $u = \pi/2$.

Now, differentiate

$$\frac{dy}{dx} = -2\csc 2x,$$

which comes from combining the formula for the derivative of $\cot u$ with that of $u = 2x$ by the **chain rule**.

Next, evaluate at $x = \pi/4$, which gives $2x = \pi/2$, to get that the slope of the tangent line is -2, and the equation of the tangent line (in **point-slope form** is

$$(y - 0) = -2\left(x - \frac{\pi}{4}\right).$$

Simplification is not required.

Exercise 36 The instructions are to gather information that can be used to sketch a graph of

$$y = x - \sin x$$

for $0 \le x \le 2\pi$. This requires the first and second derivatives:

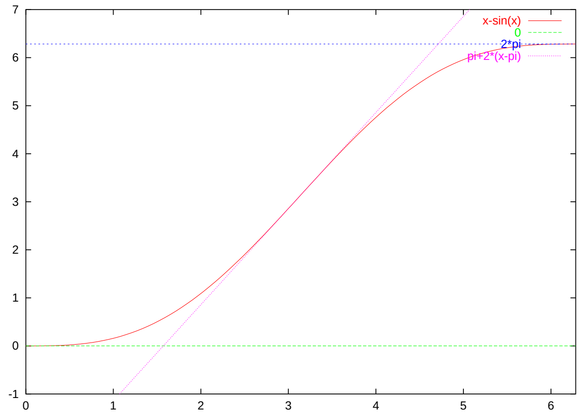
$$\frac{dy}{dx} = 1 - \cos x$$
$$\frac{d^2y}{dx^2} = \sin x$$

First, consider the first derivative. A fundamental property of trigonometric functions is $-1 \le \cos x \le 1$, so

$$0 \le \frac{dy}{dx} \le 2.$$

The graph must then be **increasing**, although it has **critical points** where $\cos x = 1$. These are at the **integer multiples** of 2π (when the point drawing a circle gets back to its starting position on the **positive** x axis). For the given interval, this happens only at the endpoints $x = 0$ and $x = 2\pi$.

Now, consider the second derivative. Inflection points can occur only where the second derivative (if it exists) is zero. For this graph, that requires $\sin x = 0$. Such values are the **integer multiples** of π (when the point drawing a circle gets back to the **whole** x axis). In addition to the endpoints, this allows $x = \pi$. For $0 < x < \pi$, $\sin x > 0$ and the curve is **concave upward**; for $\pi < x < 2\pi$, $\sin x < 0$ and the curve is **concave downward**. To check this information, here is a plot produced by a graphing program including a sketch of the requested function together with the tangent lines at the **critical points** and at the **point of inflection**. Note how much of what is **seen** in the graph is given by the way that it relates to these three lines.


x-sin(x)
0
2*pi
pi+2*(x-pi)
7
6
5
4
3
2
1
0
-1
0
1
2
3
4
5
6