Section 5.2 Some exercises exploit

$$y = \log_b x \iff x = b^y$$

to express statements in terms of logarithms instead of exponentials.

Exercise 4 Given

$$5^{-3} = \frac{1}{125},$$

the logarithmic form is

$$\log_5\left(\frac{1}{125}\right) = -3.$$

Exercise 10 Given

$$16^{-1/4} = 0.5,$$

the logarithmic form is

$$\log_{16} 0.5 = -1/4.$$

Exercise 18 The word **simplify** is overworked in algebra and precalculus. The typical meaning is, "use the formula you just learned to transform the following expression, and **pretend** that it is simpler". If you use a **Computer Algebra System** (e.g., **Maple**), you will find that it has an instruction named simplify, but often it returns the original expression unchanged. It is as if the system is saying, "I think this is already simple. What more do you want?" Sometimes you can force a transformation by using a more precise term like **expand** or **factor** (one of which which may undo the effect of the other), or modify the **simplify** instruction to indicate the type of transformation you want. In this case, the intent is to replace logarithms of **products**, **quotients**, or **powers** by **linear combinations** of the logarithms of the components of such expressions. These are likely to be simpler expressions. Thus, replacing $\log ab$ by $\log a + \log b$ transforms

$$\log\left(x(x^2+1)^{-1/2}\right)$$

into

$$\log x + \log\left((x^2+1)^{-1/2}\right).$$

Then replacing $\log a^c$ by $c \log a$ in the second term gives

$$\log x - \frac{1}{2}\log(x^2+1).$$

Exercise 34 The exercise asks to **use logarithms to solve**

$$\frac{1}{3}e^{-3t} = 0.9.$$

This means, "first solve for an exponential, then take a logarithm to find the exponent before continuing algebraic methods". Here are some steps:

$$\frac{1}{3}e^{-3t} = 0.9$$

$$e^{-3t} = 2.7$$

$$-3t = \ln 2.7$$

$$t = -\frac{1}{3}\ln 2.7 \approx -0.3311$$

Exercise 40 Another given equation with the steps needed to solve it.

$$\begin{aligned}
\frac{200}{1+3e^{-0.3t}} &= 100 \\
2 = \frac{200}{100} &= 1 + 3e^{-0.3t} \\
3e^{-0.3t} &= 2 - 1 = 1 \\
e^{-0.3t} &= \frac{1}{3} \\
-0.3t &= \ln\left(\frac{1}{3}\right) = -\ln 3 \\
t &= \frac{\ln 3}{0.3} \approx 3.662
\end{aligned}$$

Section 5.5 These exercises deal with derivatives of expressions involving logarithms. Typically, a function $f(x)$ will be given and $f'(x)$ is to be found.

Exercise 2

$$\begin{aligned}
f(x) &= \ln(5x) \\
f'(x) &= \frac{1}{5x} \cdot \frac{d}{dx}(5x) = \frac{1}{5x} \cdot (5) = \frac{1}{x}
\end{aligned}$$

Are you surprised? $\ln(5x)$ and $\ln x$ have the same derivative. You shouldn't be.

$$\ln(5x) == \ln 5 + \ln x,$$

so the two expressions differ by a constant.

Exercise 4

$$\begin{aligned}
g(x) &= \ln(2x+1) \\
g'(x) &= \frac{1}{2x+1} \cdot \frac{d}{dx}\left(2x+1\right) = \frac{2}{2x+1}
\end{aligned}$$

Exercise 8

$$\begin{aligned}
f(x) &= \ln\left(\sqrt{x}+1\right) \\
f'(x) &= \frac{1}{\sqrt{x}+1} \cdot \frac{d}{dx}\left(\sqrt{x}+1\right) \\
&= \frac{1}{\sqrt{x}+1}\left(\frac{1}{2}x^{-1/2}\right) \\
f'(x) &= \frac{1}{\sqrt{x}+1} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\left(x+\sqrt{x}\right)}
\end{aligned}$$