Math 351 Section 2 Workshop 12

April 18, 2022

Warm-up 1.
Let $G = \mathbb{Z}_{15}$ and let $H = \langle 3 \rangle$.

(a) List the elements of H.

(b) Show that H is a normal subgroup of G.

(c) Describe the quotient group G/H. (Show that it isomorphic to a different, familiar group.)

Problem 2.
Give an example of a normal subgroup N in a group G and elements $n \in N$, $g \in G$ such that $ng \neq gn$.

Problem 3.
Let G be a group.

(a) Show that $\langle e \rangle$ and G are each normal subgroups of G.

(b) Show that $G/\langle e \rangle \cong G$ and that G/G is a group with one element.

Problem 4.
Let K be the subgroup of S_3 generated by the element (123).

(a) List the elements of K.

(b) Is K a normal subgroup of S_3? If so, describe S_3/K.

Problem 5.
If $f : G \to H$ is a surjective group homomorphism and N is a normal subgroup of G, prove that $f(N)$ is a normal subgroup of H.

Problem 6.
Let H be the subgroup $\langle 6 \rangle$ in $\mathbb{Z}_{18}$. Show that $\mathbb{Z}_{18}/H \cong \mathbb{Z}_6$.

Problem 7.
Prove that $\mathrm{SL}(2, \mathbb{R})$ is a normal subgroup of $\mathrm{GL}(2, \mathbb{R})$.

Problem 8.
If G is a cyclic group, prove that G/N is cyclic for any subgroup N of G.

Problem 9.
Show that every element of the additive group $\mathbb{Q}/\mathbb{Z}$ has finite order.