Countable Universal Graphs with Forbidden Subgraphs

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Universal Theories and Combinatorics

- Universal Theories
  - Permutation Pattern Classes
  - Joint embedding (Ruskuč, Braunfeld)
- Universal graphs (Rado, Komjáth, Pach)
  - Existentially complete structures
  - Model Completeness
  - Smallness

An Algorithmic Problem

- Example: Acyclic graphs.
- The nice case: Finitely many forbidden finite subgraphs.
- Decision problems for T^*_φ: j.e.p., small, $\aleph_0$-categorical, stable, etc.
  - Forbidden induced graphs.
  - Forbidden Subgraphs
    - Focus $\aleph_0$-categorical case

Review and Examples

Problem: Existence of universal $\mathcal{F}$-free graphs.

$\mathcal{F}$ finite set of finite, connected graphs.
$T^*_{\mathcal{F}}$: Complete, model-complete theory.

- When is it small?
- When is is $\aleph_0$-categorical?

Examples

- Forbidden tree
- Homomorphism-closed
- Forbidden cycles
- 2-connected constraints

One constraint: Conjectures

a. Solidity
b. Block path conjecture
c. Decidable: explicitly
d. Small iff $\aleph_0$-categorical except for near paths.

Theorem (ChSh 2016)

$(b) \implies (a)$

Conjecture

For minimal block size at least 4: clique, 2-bouquet, or type $(m, 4, n)$.
For minimal block size at least 3: As on next page.

Minimal block size 3

ℓ	Form
(general)	$(3^{\ell-1}, n)$; or $(3^{\ell-2}, n, 3)$ or $(3^{\ell-2}, 4, 4)$
2	$(4, n)$, or $(5, n)$ with $n \geq 6$
3	(n_1, m, n_3) with $m = 3$ or 4
4	$(n_1, 3, 3, n_4)$ with $n_4 \geq n_1 + 2$
"	$(3, n, 3, n)$ with $n > 4$
"	$(3, 4, 4, 4)$
"	$(3, 4, 3, n)$ $(4, 4, 3, n)$ with $n \geq 4$
5	$(4, 4, 3, 3, n)$ with $n \geq 9$
"	$(3, n_2, 3, 3, n_5)$ with $n_2, n_5 \geq 4$ and $\lvert n_2 - n_5 \rvert \geq 2$
"	$(3, 3, n, 3, n)$ with $n \geq 5$

Methods

- acl
- Pruning

$\aleph_0$-categoricity: acl is locally finite
Pruning: remove block leaves; variations

Open Problems

Problems (Positive Cases)

Prove $\aleph_0$-categoricity for the following cases.

- Clique with whiskers
- Chain of triangles

Problem (Forbidden subgraphs)

When is the theory $T^*_{\mathcal{F}}$ stable?

Problems (Permutation Pattern Classes)

- j.e.p.
- Model Complete?
- Small
- $\aleph_0$-categorical?