Linear algebra formulas and algorithms

Rotation Matrix: $A_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Linear systems

Consistency of $A\mathbf{x} = \mathbf{b}$: $\mathbf{b}$ is in the column space
LU: $U = E_k \cdots E_1 A$: $L = E_1^{-1} \cdots E_k^{-1}$
Back and forth substitution: $L\mathbf{y} = \mathbf{b}; U\mathbf{x} = \mathbf{y}; LU\mathbf{x} = \mathbf{b}$
Cramer: $x_i = \det(B_i)/\det(A)$.

Matrix algebra

$(AB)^{-1} = B^{-1}A^{-1}$ $\quad$ $(P^{-1}DP)^n = P^{-1}D^nP$ $\quad$ $\det(AB) = \det(A)\det(B)$

Inversion Algorithm: $[A \quad I] \to [I \quad A^{-1}]$

Invertibility: $\det A \neq 0$

Basis algorithms

1. Row space: nonzero rows of row reduced form. Dimension: rank.
2. Column space: pivot columns of original matrix. Dimension: rank.
3. Null space: one basis vector per free variable: set one free variable equal to 1, set the others equal to 0, and solve for the basic variables. Dimension: nullity.
4. Eigenspace: Null space of $A - \lambda I$.
5. Orthogonal complement of column space: Null space of transpose.
6. Orthogonal basis: Gram-Schmidt applied to any known basis.

Eigenvalues

Roots of the characteristic polynomial

Sum: trace. Product: determinant.

Diagonalization and Power algorithms; exponential

$P^{-1}AP = D$: P is the matrix of eigenvectors, D is the matrix of eigenvalues.
Symmetric case: $P^TP = 1$ (use an orthogonal basis consisting of unit vectors).
$A^n = PD^nP^{-1}$; $e^A = Pe^DP^{-1}$; $\det(e^A) = e^{\text{tr}}(A)$
Conic sections: diagonalize the matrix of the quadratic form and look at the conic section corresponding to the diagonal matrix.

Dot products and Projections

$||\mathbf{u}|| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$
$A\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot A^T\mathbf{v}$ (adjoint property)
Projection of $\mathbf{v}$ in the direction of $\mathbf{u}$: $\dfrac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u}$
The projection matrix in this case is $\mathbf{v}\mathbf{v}^T/\mathbf{v}^T\mathbf{v}$ (a matrix divided by a number).
Projection onto a subspace V: if you have an orthogonal basis for V, use the sum of the projections onto each basis vector; otherwise, use $P = A(A^TA)^{-1}A^T$ where the columns of A are a basis for V (laborious).
Gram-Schmidt: remove from each vector its projection onto the span of the previous vectors; use the *new* basis vectors to do this.
Least squares ($A\mathbf{x} = \mathbf{b}$): project $\mathbf{b}$ into the column space of A and solve; in practice, multiply on the left by A^T and solve.