

## Some useful formulas in and around linear algebra

### Rotations

$$A_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

Values of Trigonometric functions

$\theta$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

### Linear systems

Consistency of  $A\mathbf{x} = \mathbf{b}$ :  $\mathbf{b} \in \text{Span}(\mathbf{a}_1, \dots, \mathbf{a}_n)$ .

Back and forth substitution:

$$L\mathbf{y} = \mathbf{b}; U\mathbf{x} = \mathbf{y}; LU\mathbf{x} = \mathbf{b}$$

$$U = E_k \cdots E_1 A; L = E_1^{-1} \cdots E_k^{-1}$$

Elementary row operations: multiply by elementary matrices on the left.

### Matrix algebra

$$(AB)^T = B^T A^T$$

$$A \cdot (c_1 \mathbf{u}_1 + c_2 \mathbf{u}_2) = c_1 A\mathbf{u}_1 + c_2 A\mathbf{u}_2$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

$$(A^T)^{-1} = (A^{-1})^T$$

Inversion Algorithm:  $[A \quad I] \rightarrow [I \quad A^{-1}]$

To write an invertible matrix  $A$  as a product of elementary matrices: (1)  $E_k \cdots E_1 A = I$  (row reduction to reduced row echelon form); (2)  $A = E_1^{-1} \cdots E_k^{-1}$ .

### Linear Correspondence Principle

Any linear relations among the *columns* of a matrix are unaffected by elementary *row* operations.