History of Mathematics in Mesopotamia

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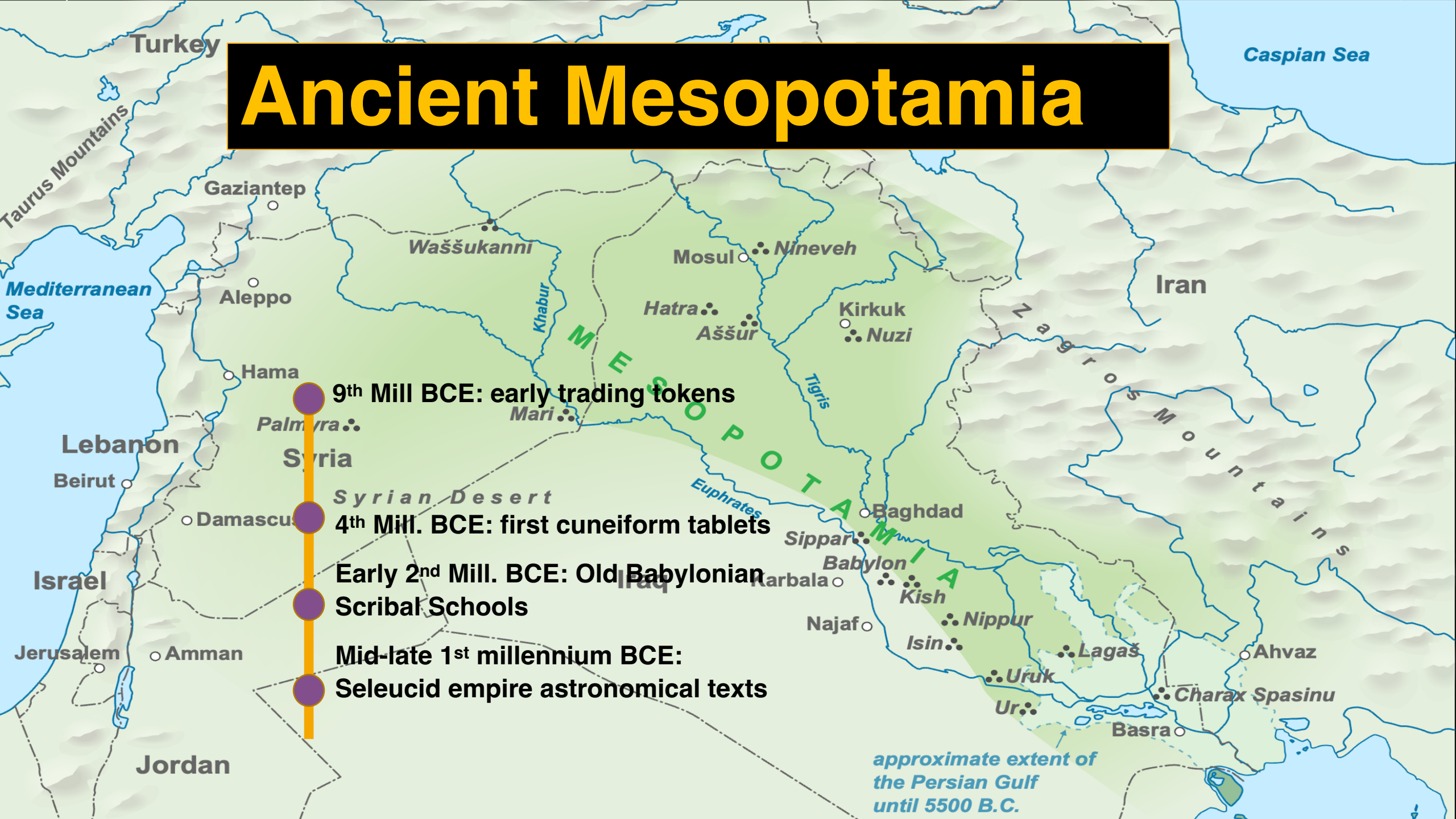
Ancient Mesopotamia



- 9th Mill BCE: early trading tokens
- 4th Mill. BCE: first cuneiform tablets
- Early 2nd Mill. BCE: Old Babylonian Scribal Schools
- Mid-late 1st millennium BCE: Seleucid empire astronomical texts



Tokens & Bullae

- Earliest finds c. 8000 BCE
- Used widely across Mesopotamia and as far back as Iran
- Tokens represented livestock quantities
- Sealed in an envelope and “signed” with a stamp or cylinder seal
- Continued to be used through second millennium BCE
- Later adaptation for document envelopes



Fig 5. Envelope from Susa, Iran, ca. 3300 BC, Courtesy Denise Schmandt–Besserat with permission from Mus&eacutee du Louvre, D&eacutepartement des Antiquit&eacutes Orientales. The lenticular disks each stand for "a flock" (—10 sheep?) . The cones represent small measures of grain.



Fig 6. Impressed tablet from Godin Tepe, Iran, ca. 3200 BC. Courtesy Denise Schmandt–Besserat, with permission from Cuyler Young Jr., Royal Ontario Museum, Toronto. The circular signs, deriving from the sphere token, stand for 1 large measure of grain. The wedges, deriving from the cone token, for a small measure of grain.

Researched by Denise Schmandt-Basserat:

https://sites.utexas.edu/dsb

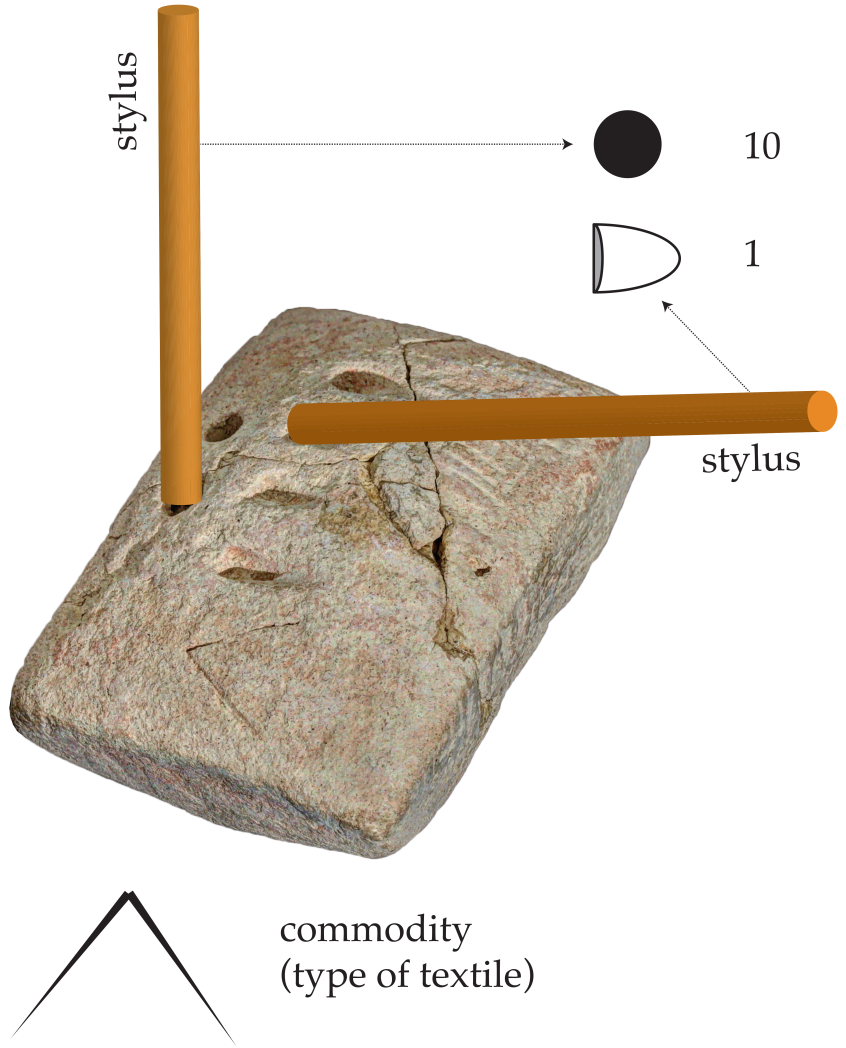
The History of Counting **(1999)**

Uruk IV
(3350-3200 BCE)

- Some of earliest inscribed tablets found at Uruk
- 24 units of a textile
- Number inscribed with a round stylus - archaic
- Faint seal impression on the obverse, reverse and edge
- Not found in their original context, but later as floor filler



YPM BC 034486

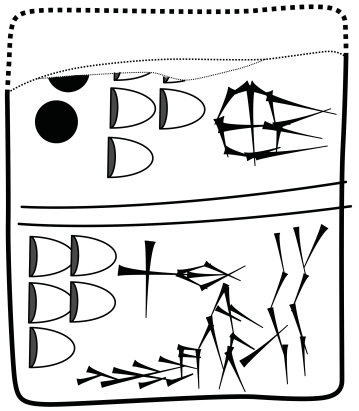


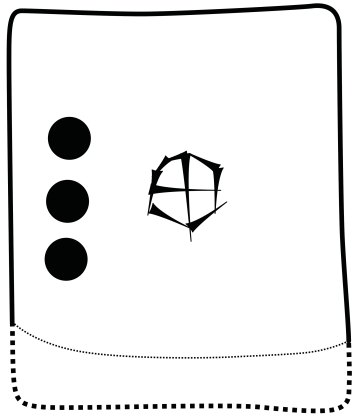
Uruk III
(c. 3100 BCE)

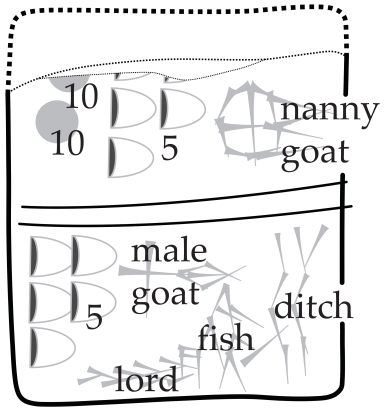
- About 100 years after earliest accounting (“writing”) tablets found in Uruk
- Gradual shift from pictographic to cuneiform signs
- Counting systems specific to items counted: circles and semi-circles for humans, animals, containers

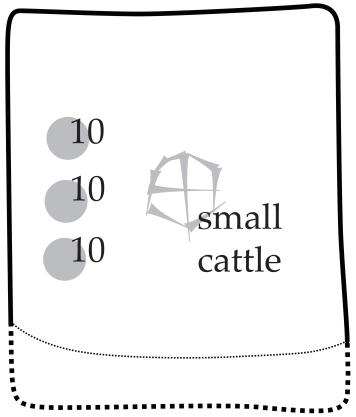












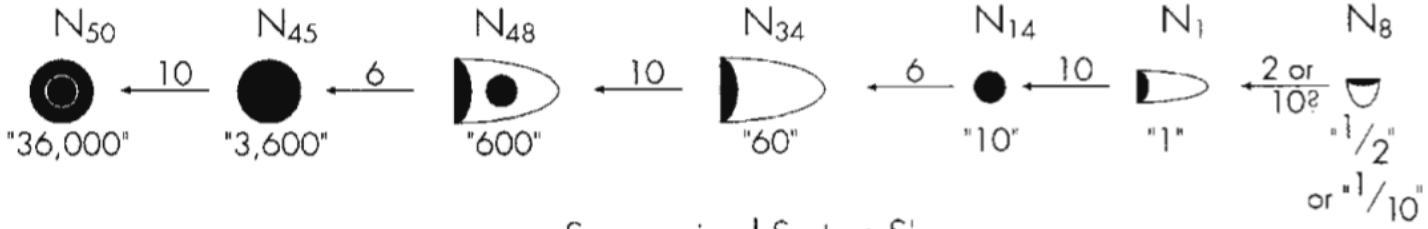
25 nanny goats

5 male goats
(under the responsibility of) the (official written) EN KU_6 RAD

(Total:)
30 small cattle

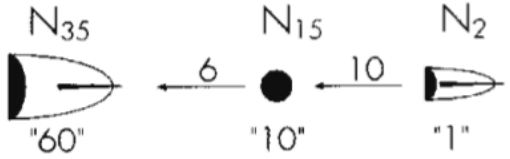
YBC 7056

Sexagesimal System S



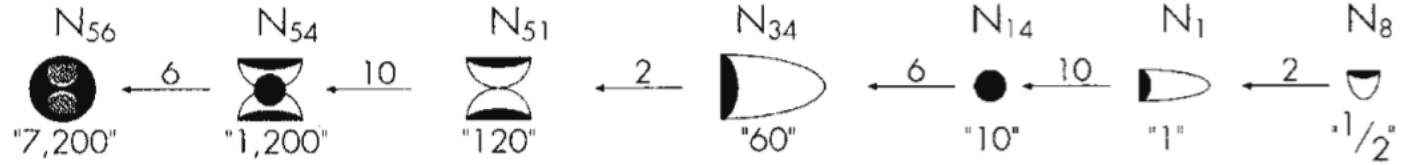
System used to count most discrete objects, for example, humans and animals, dairy and textile products, fish, wooden and stone implements,and containers

Sexagesimal System S'



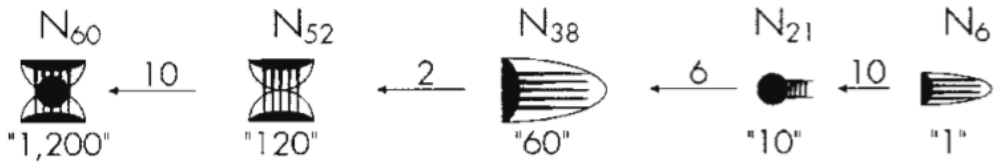
System derived from the sexagesimal system, used to count certain objects, for example, dead animals from herds and jars of certain types of liquids

Bisexagesimal System B



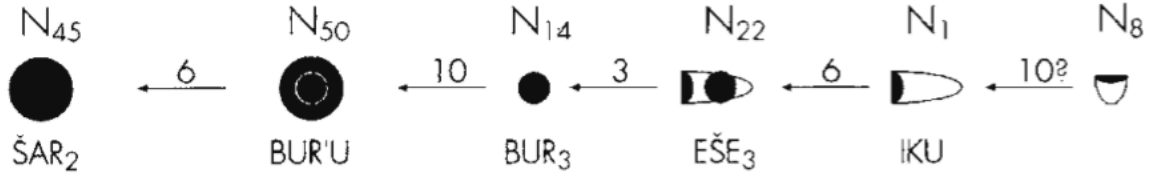
System used to count discrete grain products, cheese, and fresh fish; all objects noted with this system appear to belong to a rationing system

Bisexagesimal System B*



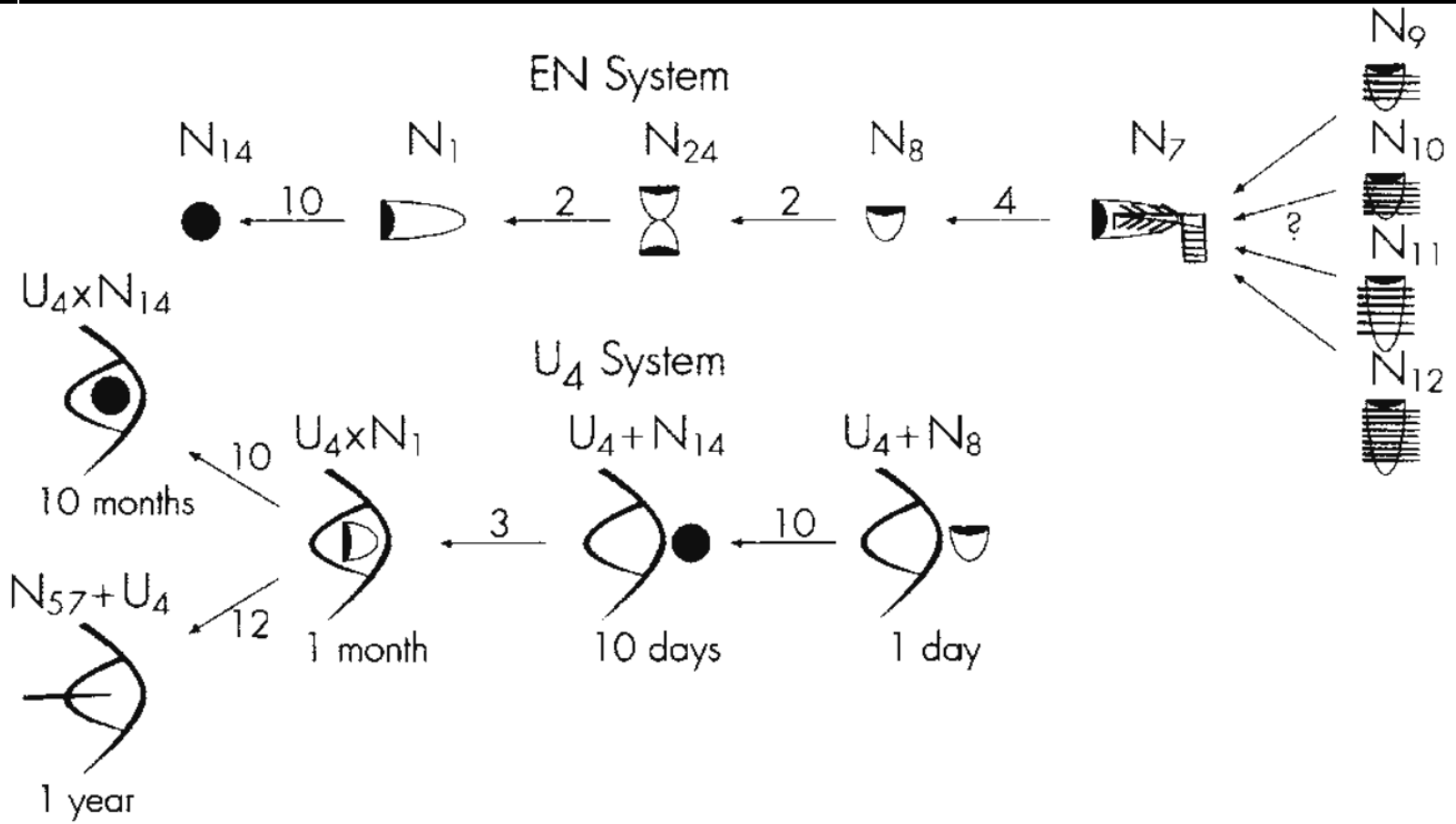
System derived from the bisexagesimal system, used to count rations of an unclear nature, possibly a type of fish

GAN_2 System



System used to note area measures

Woods, *Visible Language* (2010, 41)



System attested only in the Uruk IV period with uncertain application; possibly used to note weight measures

System used to note time and calendar units (twelve 30-day months to a year)

Woods, *Visible Language* (2010, 41)
Sign graphics by Klaus Wagensonner

Sumerian:
šid “to count”
sagga “accountant”
umbisang scribe”



Chinese:
suan4
“to calculate”



Interest

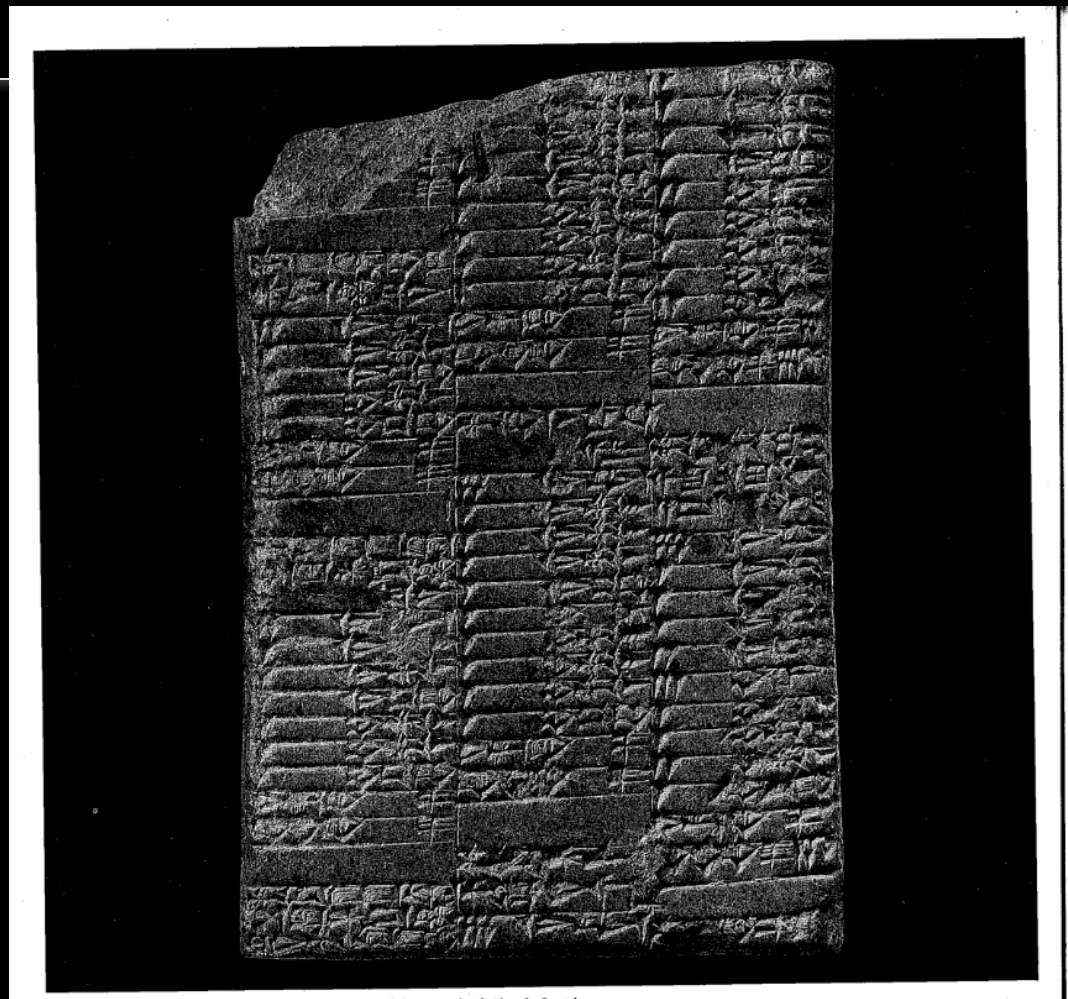
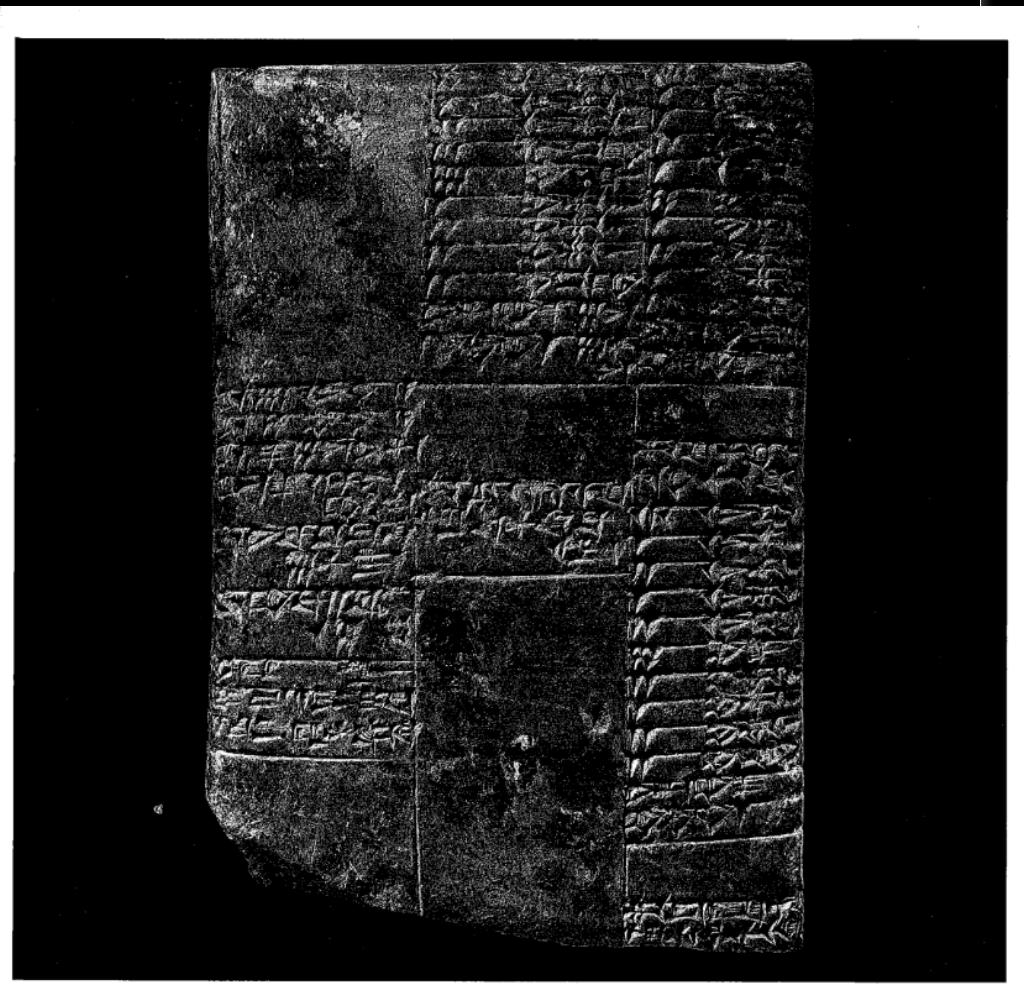
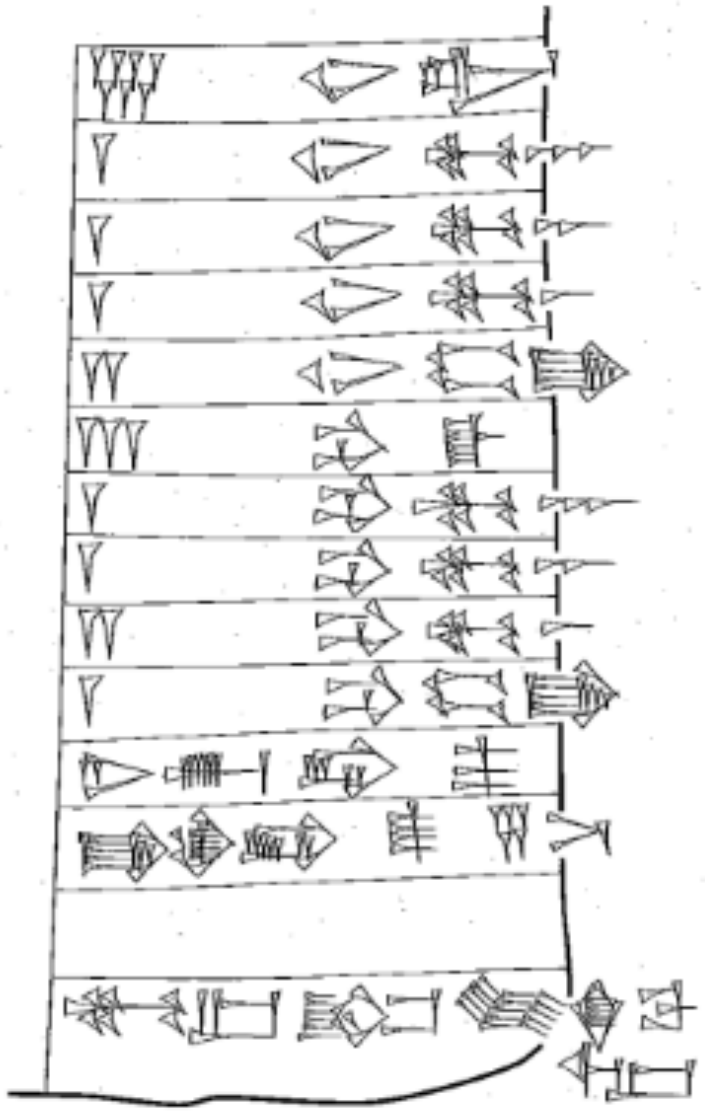


Figure 76. Neo-Sumerian tablet with a calculation of the growth of a herd of cattle.



Ur III (late 3rd M) tablet ; AO 5499. Nissen, Damerow & Englund, *Archaic Bookkeeping* (1993, 97)



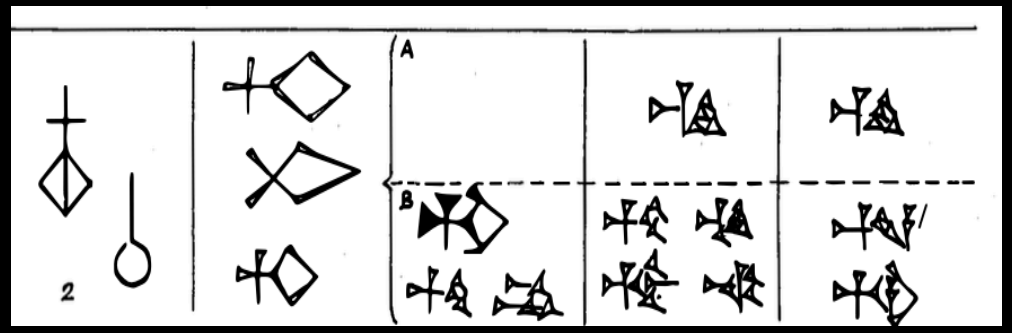
obverse, bottom
right corner

7	áb máḫ
1	áb mu.3×AŠ
1	áb mu.2×AŠ
1	áb mu.1×AŠ
2	áb amar.ga
3	gu_4 gal
1	gu_4 mu.3×AŠ
1	gu_4 mu.2×AŠ
7	gu_4 mu.1×AŠ
1	gu_4 amar.ga
ì.nun.bi	3 (bán)
ga.àr.bi	4 (bán) 5 sìla
mu Ur.bí.lum^{ki} ba.ḫul	

7	full-grown cows,
1	three-year-old cow,
1	two-year-old cow,
1	one-year-old cow,
2	sucking heifer calves,
3	full-grown bulls,
1	three-year-old bull,
1	two-year-old bull,
7	one-year-old bulls,
1	sucking bull calf,
the dairy fat involved:	3 (bán),
the cheese involved:	4 (bán) 5 sìla,
Year: "Urbilum was destroyed" (45th regnal year of Šulgi)	

Figure 77. Excerpt from the text illustrated in figure 76, stating the number of cattle registered in the seventh year of the model calculation.

Interest



máš (Sumerian “goat”)
ṣibtum (Akkadian “interest”)

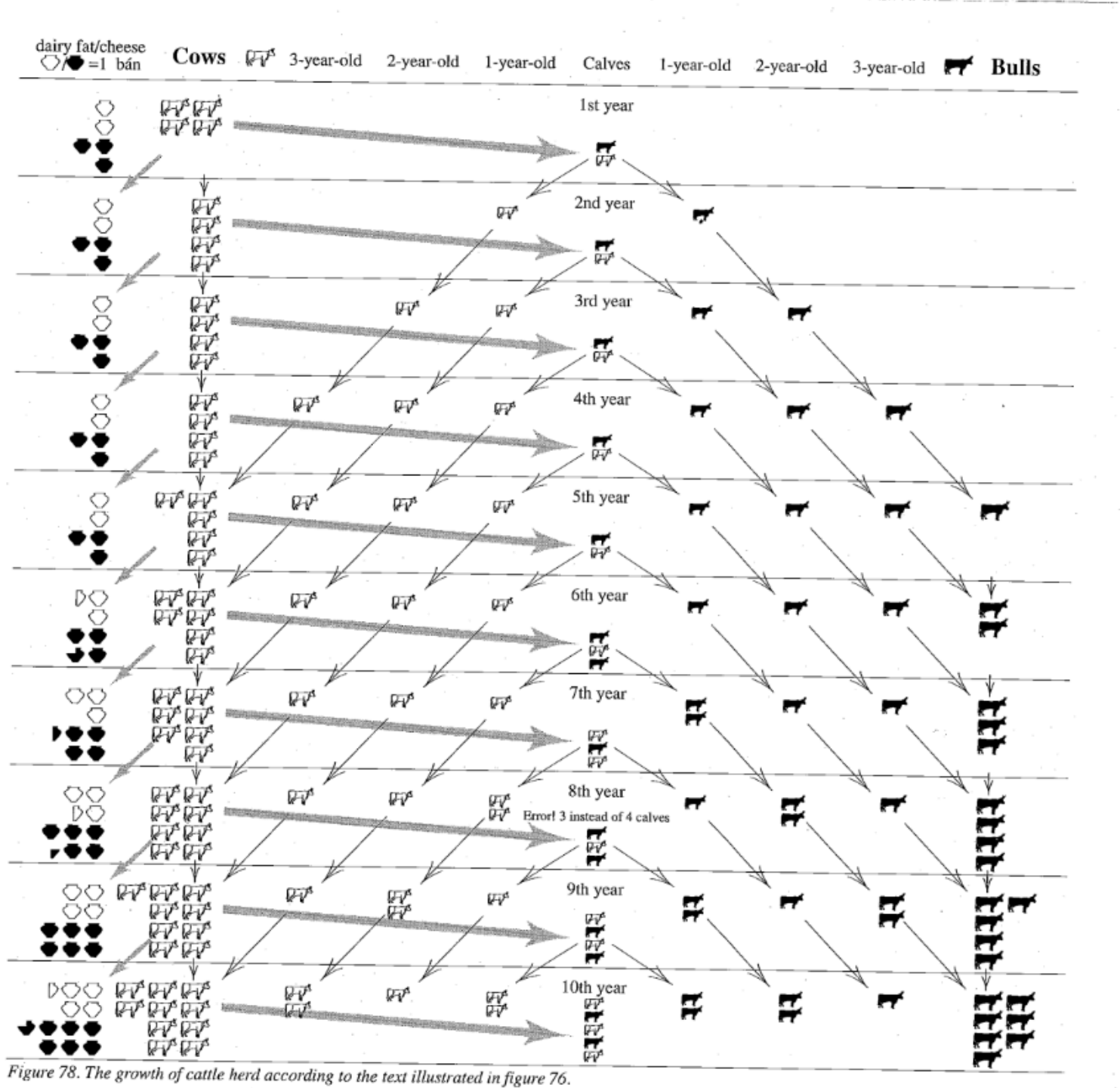


Figure 78. The growth of cattle herd according to the text illustrated in figure 76.

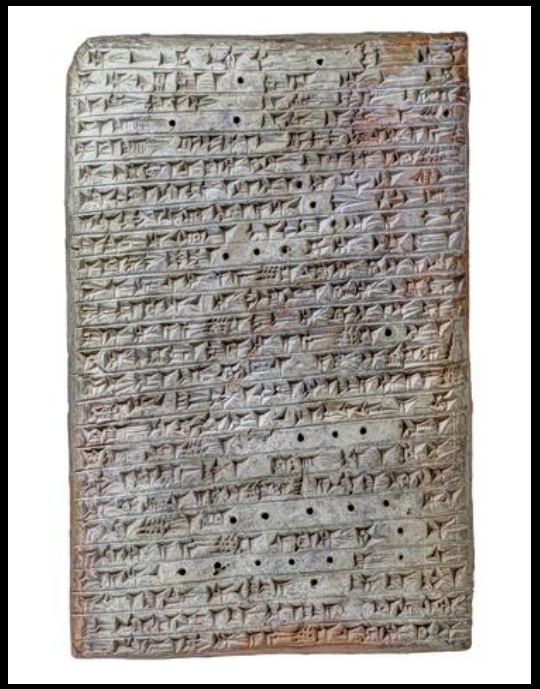
Cuneiform



YBC 7056
(Uruk III/ 3100)



YBC 8633 (OB/
early 2nd M.)



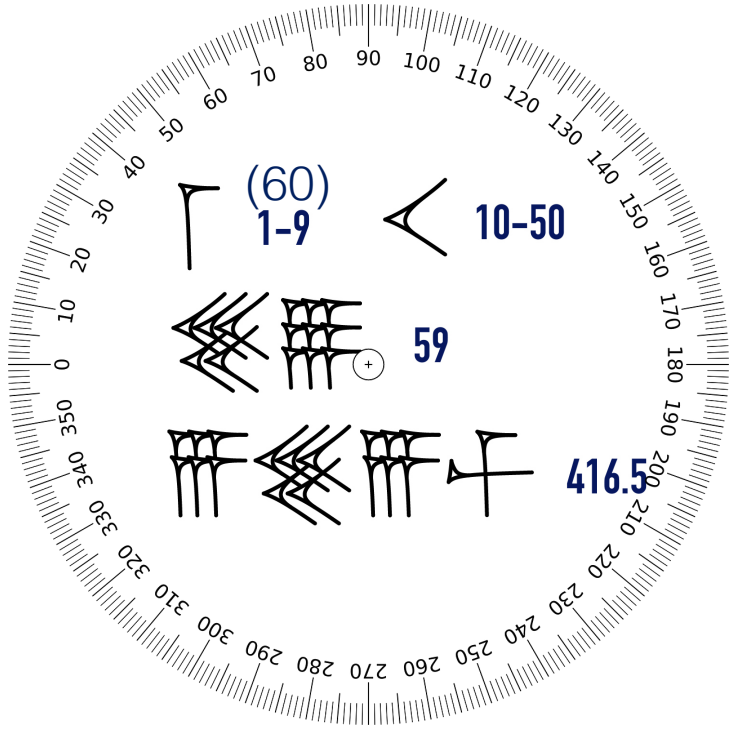
YBC 02246
(MB/mid 2nd M.)

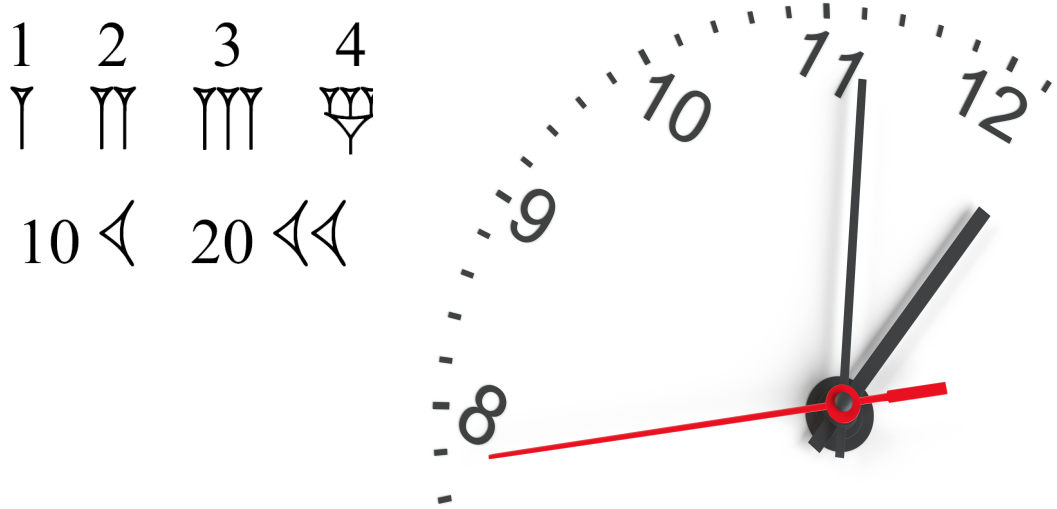


MLC 01880
(Hellenistic/ from4th C.)

Old Babylonian Math

- Sexagesimal system
- Vertical wedges: 1-9
- Winkelhaken: 10 – 50
- Vertical wedges again: units of 60
- Can you think of sexagesimal systems used today?





Scribal Education

- Small private settings
- House F in Nippur (Stone 1978) c. 127 math tablets
- Most of the Old Babylonian math tablets found in scribal settings
- Often exercises with a clean teacher version and a messy student version

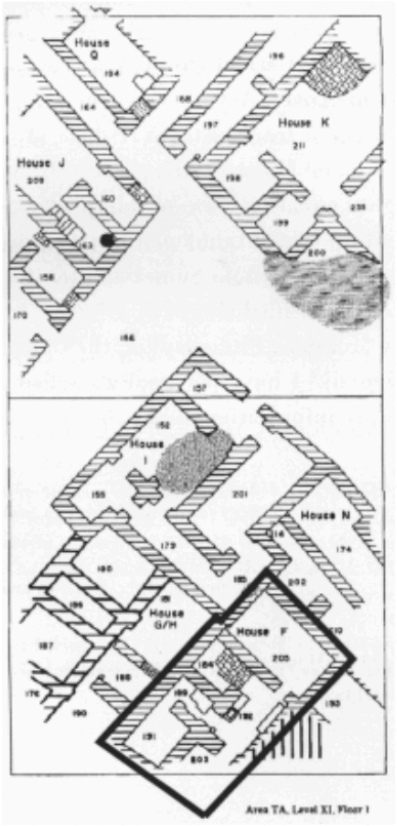


Fig. 2. — Excavation plan of Area TA (after Stone 1987: pl. 19)

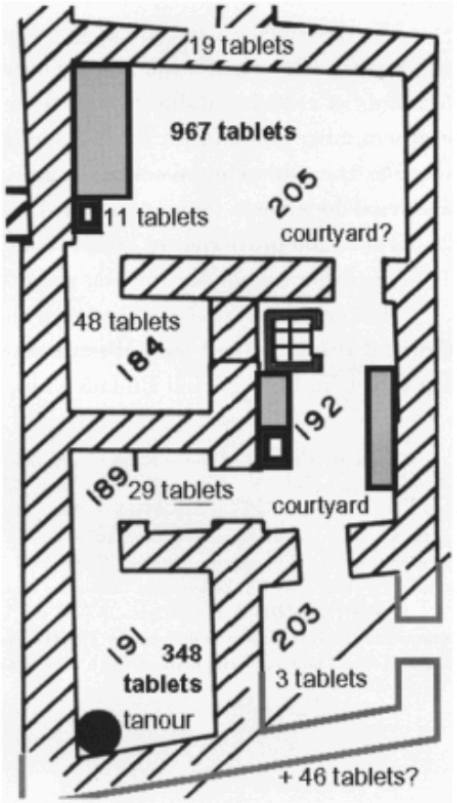


Fig. 3. — Composite excavation plan of House F, Level 10 (after Stone 1987: pls. 17–19)



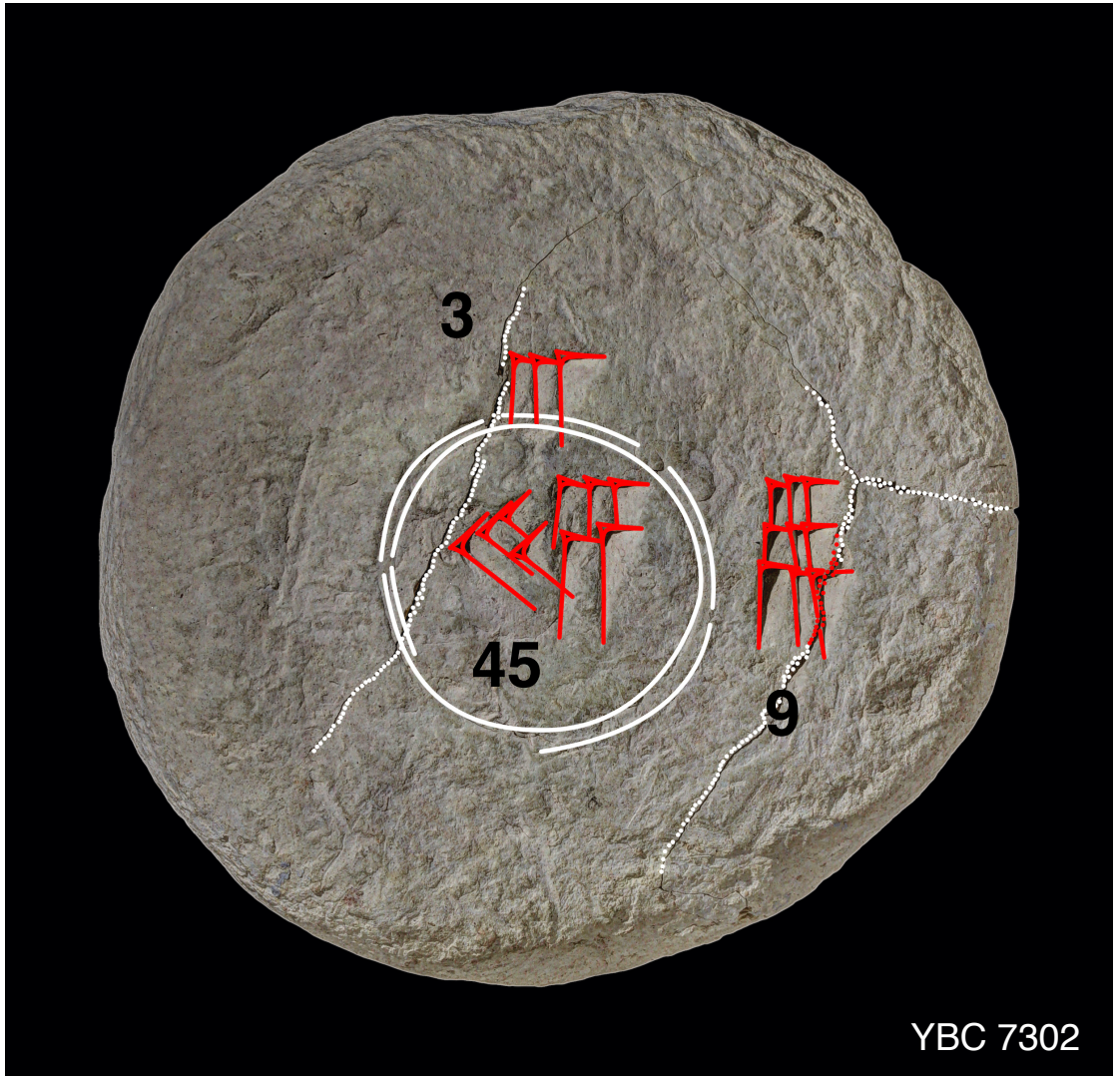
YPM BC 021354

Square Root of 2

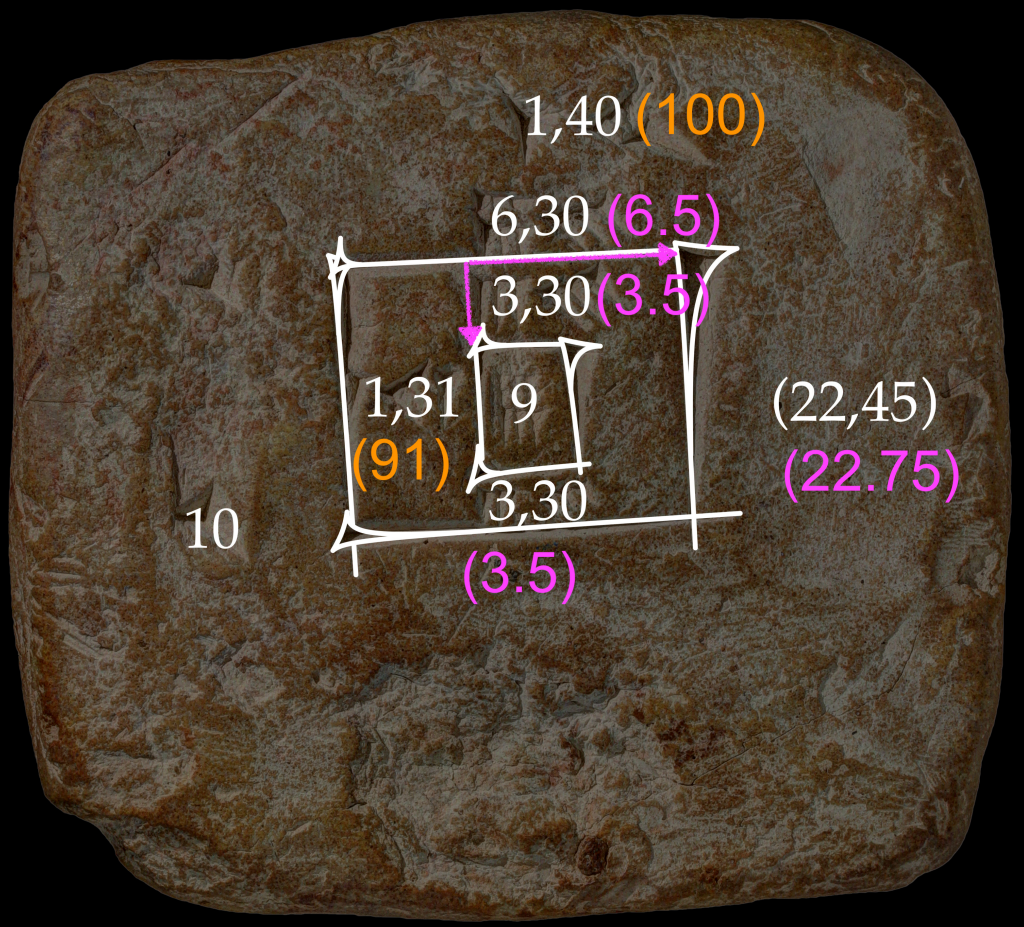
- 1, 24, 51,10 which represents $1+(24/60)+(51/60^2)+(10/60^3) = 1.41421296$.
- $d= \sqrt{a^2+a^2}= \sqrt{2}a$ *(42.426 = √2x30)*
- Demonstrates awareness of a constant relationship between sides and the hypotenuse of a right triangle
- C. 1300 years before Pythagoras
- Similar pattern seen in “Plimpton Triplets” OB tablets at Columbia
- Likely a scribal exercise from Larsa
- Round “lentil” shape typical for school exercises

$A = \pi r^2$

- Sexagesimal system: 3/60,9/60, 45/60,
- the circumference was squared, then multiplied by the rough coefficient of 5/60
- $A = \pi r^2 = C^2/4\pi \approx C^2/12$
- Approximates the value of π to 3







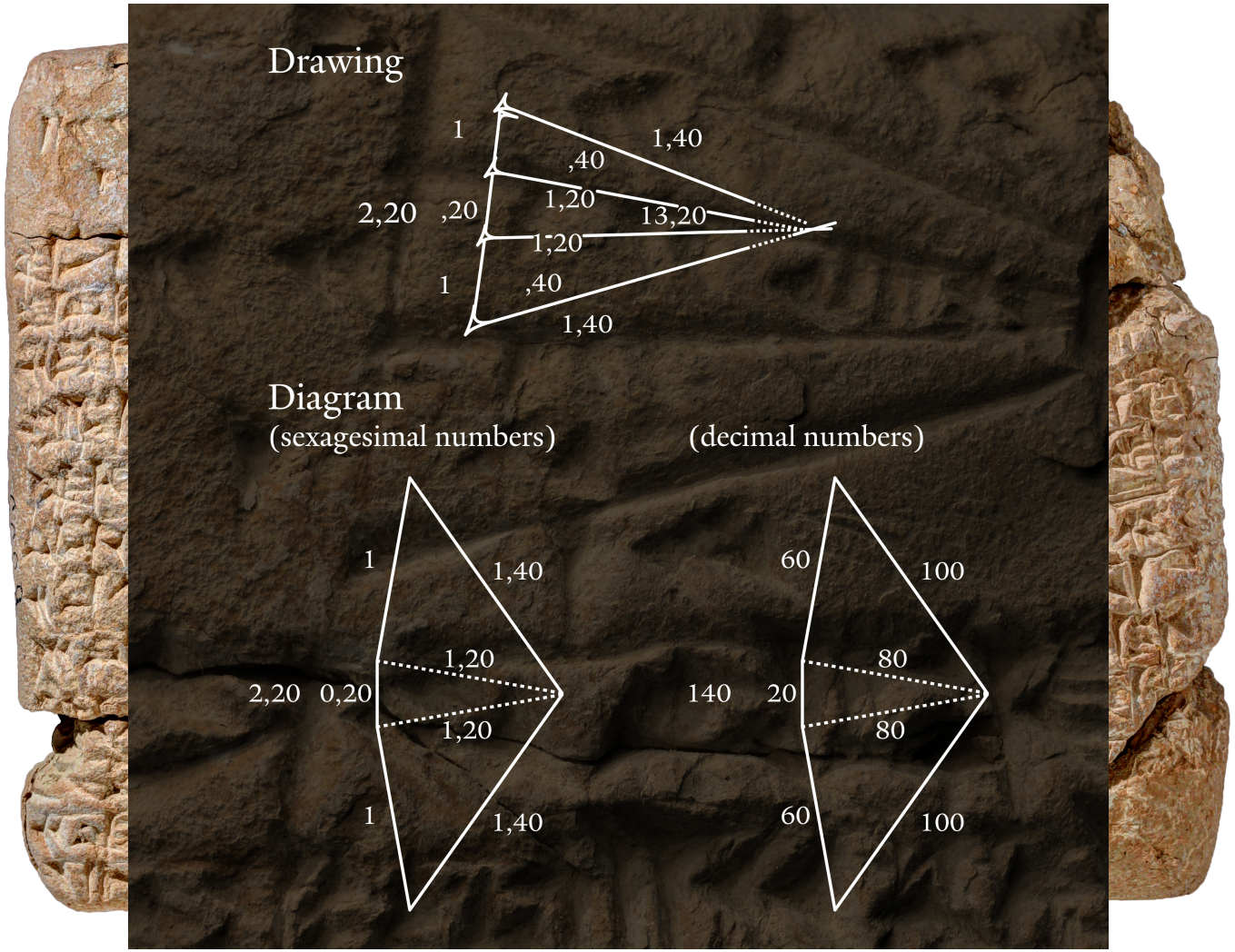
(10 x 10) - 9 = 91

3.5 x 6.5 = 22.75 | 22.75 x 4 = 91

YBC 7359

Isosceles triangle

- Mix of narrative and illustrated instructions
- 2nd person instructions “atta” (you), also used in recipe tablets
- How to partition an isosceles triangle
- Total figure slightly different from expectation – different interpretations



YBC 08633

Applied mathematics

- Question and answer format
- Several sections separated by a double ruling
- Lines 1-2 deal with interest, 3-13 with commerce, 14-16 with price equivalency and line 17 with working out original quantity.
- Example: “1 gur of common oil, 9 sila per shekel I bought. 7 ½ sila per shekel I sold. The profit is how much? 6 ⅔ shekels is the profit.”
- *Gur = c. 250 liters/ 300 sila*
- *Shekel (gín) = c. 8g (of silver) as payment*



Resources

Ancient Mesopotamia Speaks, Edited by Agnete W. Lassen, Eckart Frahm, and Klaus Wagensonner (2019)

Yale Babylonian Collection Instagram Math Series: https://www.instagram.com/stories/highlights/17905640299739259/?hl=en

Mesopotamian Mathematics: Some Historical Background in Using History to teach Mathematics: an International Perspective, Katz, Victor ed. (2000)

http://oracc.museum.upenn.edu/dccmt/downloads/robson-katz-2000.pdf

Mathematics in Ancient Iraq, by Eleanor Robson (2008)