Trig: In a right triangle:

\[
\sin \theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos \theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan \theta = \frac{\text{opp}}{\text{adj}}, \quad \cot \theta = \frac{\text{adj}}{\text{opp}}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}.
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>(\pi/6)</th>
<th>(\pi/4)</th>
<th>(\pi/3)</th>
<th>(\pi/2)</th>
<th>(3\pi/2)</th>
<th>(2\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin x)</td>
<td>0</td>
<td>1/2</td>
<td>1/(\sqrt{3})</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>(\cos x)</td>
<td>1/(\sqrt{3})</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Periodicity: \(\sin(x + 2\pi) = \sin(x)\), \(\cos(x + 2\pi) = \cos(x)\), \(\tan(x + \pi) = \tan(x)\).

Identities: \(\sin^2 x + \cos^2 x = 1\), \(1 + \tan^2 x = \sec^2 x\), \(\sin(2x) = 2 \sin x \cos x\), \(\cos(2x) = \cos^2 x - \sin^2 x\).

Addition: \(\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y\), \(\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y\), \(\pi \approx 3.1416\).

Inverses: The range of \(\sin^{-1} x\), \(\tan^{-1} x\) and \(\csc^{-1} x\) is the subset of \([-\pi/2, \pi/2]\) avoiding values that correspond to \(x = \infty\). The range of the other inverse trig functions is a similar subset of \([0, \pi]\).

Exponentials and logarithms: \(a, b, t, u, y > 0\), \(r, v, w, x\) any real numbers: \(a^{v+w} = a^v a^w\), \(a^{vw} = (a^v)^w\), \(a^{-v} = 1/a^v\), \(a^0 = 1\), \((ab)^v = a^v b^v\), \(\log_a(t) = \ln(t)/\ln(a)\). \(e^x = y\) is equivalent to \(x = \ln y\), \(e^{\ln y} = y\), \(\ln(e^x) = x\). \(\ln(tu) = \ln(t) + \ln(u)\), \(\ln(u^r) = r \ln(u)\), \(\ln(1/u) = -\ln(u)\), \(\ln(1) = 0\), \(e \approx 2.718\).

Squeeze Theorem: If \(f(x) \leq g(x) \leq h(x)\) near \(x = a\) and \(\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L\), then \(\lim_{x\to a} g(x) = L\).

Intermediate Value Theorem: If \(f\) is continuous on \([a, b]\) and \(N\) is between \(f(a)\) and \(f(b)\), there is a number \(c\) in \([a, b]\), such that \(f(c) = N\). Corollary: If \(f\) changes sign from \(a\) to \(b\), then \(f(c) = 0\) with \(c\) between \(a\) and \(b\).

Definition of the Derivative: \(f'(x) = \lim_{h\to 0} \frac{f(x + h) - f(x)}{h}\); \(f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}\).

<table>
<thead>
<tr>
<th>(f(x))</th>
<th>(f'(x))</th>
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</tr>
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<tbody>
<tr>
<td>(c, \text{ const.})</td>
<td>0</td>
<td>(a^x)</td>
<td>((\ln a)^x)</td>
</tr>
<tr>
<td>(x^r)</td>
<td>(r x^{r-1})</td>
<td>(\log_a(x))</td>
<td>(1/(\ln a \cdot x))</td>
</tr>
<tr>
<td>(e^x)</td>
<td>(e^x)</td>
<td>(\sin x)</td>
<td>(\cos x)</td>
</tr>
<tr>
<td>(\ln x)</td>
<td>(1/x)</td>
<td>(\cos x)</td>
<td>(-\sin x)</td>
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Rules of Differentiation: \( \frac{d}{dx}(cu) = c \frac{du}{dx} \), \( c \) a const., or \((cf)'(x) = cf'(x)\). \( \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx} \), or \((f+g)'(x) = f'(x) + g'(x)\). **Product Rule:**\( \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx} \), or \((fg)'(x) = f(x)g'(x) + f'(x)g(x)\). **Quotient Rule:** \( \frac{d}{dx}(u/v) = (u'v - uv')/v^2 \), or \((f/g)'(x) = (g(x)f'(x) - f(x)g'(x))/(g(x)^2)\).

Chain Rule: If \( y = f(u) \) and \( u = g(x) \), then \( \frac{dy}{dx} = \frac{du}{dx} \frac{dy}{du} \), or \((fg)'(x) = f'(g(x))g'(x)\). Replacing \( x \) by \( u \) and multiplying by \( \frac{du}{dx} \), we can apply the Chain Rule to all boxed derivative formulas. Some examples are: \( \frac{d}{dx}(u^n) = ru^{r-1} \frac{du}{dx} \), \( \frac{d}{dx}(e^u) = e^u \frac{du}{dx} \), \( \frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx} \), \( \frac{d}{dx}(\sin u) = (\cos u) \frac{du}{dx} \), \( \frac{d}{dx}(\cos u) = -(\sin u) \frac{du}{dx} \).

**Bodies in Free Fall:** If air resistance is neglected, then the height of a body in free fall near the surface of the earth is \( s(t) = s_0 + v_0 t - gt^2/2 \), where \( s_0 \) is the position at time \( t = 0 \), \( v_0 \) is the velocity at time \( t = 0 \), and \( g \) is the acceleration due to gravity with \( g = 32 \text{ft/s}^2 \) or \( g = 9.8 \text{m/s}^2 \).

**Linear or Tangent Line Approximation (or Linearization)** of \( f(x) \) at \( x = a \) is \( L(x) = f(a) + f'(a)(x-a) \).

**Newton’s Method** to approximate a solution \( r \) of \( f(x) = 0 \). Choose a point \( x_0 \) close to \( r \). Calculate the terms \( x_0, x_1, x_2, x_3, \ldots \) of the sequence defined recursively by \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \).

**Rolle’s Theorem:** Suppose \( f \) is a function that is continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a, b)\). If \( f(a) = f(b) = 0 \), then \( f'(c) = 0 \) for some \( c \) in \((a, b)\).

**Mean Value Theorem:** Suppose \( f \) is a function that is continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a, b)\). Then there is a point \( c \) in \((a, b)\) such that \( f(b) - f(a) = f'(c)(b-a) \).

**First Derivative Test:** Suppose that \( f \) is a differentiable function and \( f(c) = 0 \). (a) If \( f' \) changes sign from \( + \) to \( - \) at \( x = c \), a local maximum occurs at \( x = c \). (b) If \( f' \) changes sign from \( - \) to \( + \) at \( x = c \), a local minimum occurs. (c) If \( f' \) does not change sign at \( x = c \), neither a local maximum or minimum occurs at \( x = c \).

**Second Derivative Test:** Suppose that \( f \) is a twice differentiable function and \( f(c) = 0 \). (a) If \( f''(c) > 0 \), a local minimum occurs at \( x = c \). (b) If \( f''(c) < 0 \), a local maximum occurs. (c) If \( f''(c) = 0 \), the test fails.

**L'Hôpital’s Rule:** If \( \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \) or \( \pm\infty/\pm\infty \), then \( \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \). (Here, \( a \) may be a finite pt. or \( \pm\infty \).)

**Integration or anti-differentiation:** \( \int f(x) \, dx = F(x) + C \) means that \( F'(x) = f(x) \). Formulas can be found by reversing the differentiation formulas: \( \int x^r \, dx = x^{r+1}/(r+1) + C \), if \( r \neq -1 \) and \( \int x^{-1} \, dx = \ln |x| + C \).