Solutions to Quiz # 1 for Dr. Z.'s Number Theory Course for Sept. 26, 2013

1. (3 points) Convert 51 (written in the usual decimal notation) to to binary, in (i) sparse notation (ii) dense notation (iii) positional notation.

Sol. to 1.: $2^5 \leq 51$ but $2^6 > 51$ so

$$B(51) = 2^5 + B(51 - 2^5) = 2^5 + B(51 - 32) = 2^5 + B(19) \quad .$$

$2^4 \leq 19$ but $2^5 > 19$ so

$$B(19) = 2^4 + B(19 - 2^4) = 2^4 + B(3) \quad .$$

$2^1 \leq 3$ but $2^2 > 3$ so

$$B(3) = 2^1 + B(3 - 2^1) = 2^1 + B(1) = 2^1 + 2^0 \quad .$$

Going back:

$$B(19) = 2^4 + 2^1 + 2^0 \quad ,$$

$$B(51) = 2^5 + 2^4 + 2^1 + 2^0 \quad .$$

Ans. to 1(i): $51 = 2^5 + 2^4 + 2^1 + 2^0$

Including **all** the powers of 2 up to 2^5 and putting a 1 in front of those that show up, and a 0 in front of those that don't

Ans. to 1(ii): $51 = 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

Listing the coefficients, we get

Ans. to 1(iii): $51 = 110011$ (base 2).

Comment: Most people got it right.

2. (3 points) Find, by only using paper-and-pencil, the first three terms of the Euclid-Mullin sequence.

Sol. to 2: $M_1 = 2$, M_2 is the smallest prime dividing $M_1 + 1 = 2 + 1 = 3$, so $M_2 = 3$. M_3 is the smallest prime dividing $M_1M_2 + 1 = 2 \cdot 3 + 1 = 7$, so $M_3 = 7$.

Ans. to 2: The first three terms in the Euclid-Mullin series are $2, 3, 7$.

Comment: Most people got it right, **except**, some people listed **four terms**. Knowing how to follow instructions is **more** important than number theory. If you **must** to on, please write:

"Dear Dr. Z.: I know that all you asked for were three terms, so the answer to your question is: $2, 3, 7$. Since I **love** doing it, I can't resisit the temptation, just for fun, to do one more term: $2 \cdot 3 \cdot 7 + 1 = 43$, the smallest prime divding 43 is 43, so the fourth term is 43.

3. (4 points) Prove that there are infinitely many primes.

Sol. to 3: See wikipedia and thousands of other places on the internet.