

Solutions to the Quiz # 5 for Dr. Z.'s Number Theory Course for Oct. 24, 2013

1. (5 points) Using divisibility tests, determine for each of the following integers (written in the usual, base 10, way) whether it is divisible by (i) 7 (ii) 13. Explain!

a. 490735357707

b. 169130403130

Sol. to 1a: We break it up into blocks of three:

$$490|735|357|707$$

For (i) (divisibility by 7) we need to find

$$490 - 735 + 357 - 707 \pmod{7} .$$

Most people first found

$$490 - 735 + 357 - 707 = -595$$

and then took it $\pmod{7}$ getting zero (and some people complained that they can't use a calculator!, these are simple addition/subtraction problems!). But people who do not like to calculate could have done $\pmod{7}$ for each individual number and realized that they are all 0!

Ans. to 1a(i): 490735357707 is divisible by 7

For 1a(ii), it is the same way, but now we take it $\pmod{13}$.

First way: Again we get -495 . Since $495 \pmod{13} = 10$, it is **not divisible by** 13.

Second way: $490 \pmod{13} = 9$, $735 \pmod{13} = 7$, $357 \pmod{13} = 6$, $707 \pmod{13} = 5$
So

$$490 - 735 + 357 - 707 \pmod{13} = 9 - 7 + 6 - 5 = 3 .$$

So it is **not** divisible by 13.

Sol. to b: We break it up into blocks of three:

$$169|130|403|130$$

The alternating sum is

$$(169 + 403) - (130 + 130) = 312 .$$

Since 312 is not divisible by 7, 169130403130 is not divisible by 7, but since 312 is divisible by 13, 169130403130 is divisible by 13.

Ans. to 1b(i): not divisible by 7. **Ans. to 1b(ii):** divisible by 13.

2. (5 points)

What day of the week is

(i) Oct. 24, 2014

(ii) Oct. 24, 2015

(iii) Oct. 24, 2016

(iv) Oct. 24, 2017

(v) Oct. 24, 2018

Sol. of 2: Every non-leap year, it advances by 1 day (since $365 \pmod{7} = 1$). In a leap year (after Feb. 29), it advances by 2 days.

So, since today is Thursday, (i) Friday (ii) Sat. (iii) Monday (since 2016 is a leap year, and Oct. is after Feb. 29)) (iv) Tues. (v) Wed.