Solutions to Attendance Quiz # 6 for Dr. Z.’s Number Theory Course for Sept. 23, 2013

1. Using the recursive algorithm **directly**, find the product-of-prime-powers representation of 420

**Sol. of 1:**

The smallest prime dividing 420 is 2. $420/2 = 210$ is an integer. $420/2^2 = 210/2 = 105$ is still an integer, but $420/2^3 = 105/2$ is **not** an integer. So $p_1 = 2, a_1 = 2$ and $n' = 420/2^2 = 105$, and we have

$$L(420) = [2, 2], L(105).$$

The smallest prime dividing 105 is 3. $105/3 = 35$ is an integer, but $105/3^2 = 35/3$ is not, so

$$L(105) = [3, 1], L(35).$$

The smallest prime dividing 35 is 5. $35/5 = 7$ is an integer, but $35/5^2 = 7/5$ is not, so

$$L(35) = [5, 1], L(7).$$

Finally

$$L(7) = [7, 1].$$

Going **back**, we have

$$L(35) = [5, 1], [7, 1],$$  
$$L(105) = [3, 1], [5, 1], [7, 1],$$  
$$L(420) = [2, 2], [3, 1], [5, 1], [7, 1].$$

Or, in the usual notation

$$420 = 2^2 \cdot 3^1 \cdot 5^1 \cdot 7^1.$$

2. Use any method to find the product-of-prime-powers representation of $45^{50}$.

**Sol. to 2.** $45 = 3^2 \cdot 5$, so $45^{50} = (3^2 \cdot 5)^{50} = 3^{100} \cdot 5^{50}$. 