

**Solutions to Attendance Quiz # 20 for Dr. Z.'s Number Theory Course for Nov. 14, 2013**

1. Using the Quadratic Reciprocity Law and Rules 1-3, find (no credit for other methods)

$$\left(\frac{17}{23}\right)$$

**Sol. to 1:** By Rule 4 the **Quadratic Reciprocity Law**

$$\left(\frac{17}{23}\right) \left(\frac{23}{17}\right) = (-1)^{(17-1)(23-1)/4} = (-1)^{(16)(24)/4} = 1 \quad ,$$

so

$$\left(\frac{17}{23}\right) = \left(\frac{23}{17}\right) = \left(\frac{6}{17}\right) \quad ,$$

since  $23 \bmod 17 = 6$ .

Using rule (1)

$$\left(\frac{6}{17}\right) = \left(\frac{2}{17}\right) \cdot \left(\frac{3}{17}\right) \quad .$$

By Rule (3)

$$\left(\frac{2}{17}\right) = (-1)^{(17^2-1)/8} = (-1)^{36} = 1 \quad .$$

Again by Rule (4)

$$\left(\frac{3}{17}\right) \left(\frac{17}{3}\right) = (-1)^{(17-1)(3-1)/4} = 1^8 = 1 \quad .$$

Hence

$$\left(\frac{3}{17}\right) = \left(\frac{17}{3}\right) = \left(\frac{2}{3}\right) = -1$$

(since obviously 2 is not a quadratic residue mod 3, the only one is 1. You can also use Rule (4) again, if you wish. So Going back we have,

$$\left(\frac{17}{23}\right) = \left(\frac{23}{17}\right) = \left(\frac{6}{17}\right) = \left(\frac{2}{17}\right) \left(\frac{3}{17}\right) = 1 \cdot (-1) = -1$$

**Ans. to 1:**  $\left(\frac{17}{23}\right) = -1$ , in other words 17 is a **quadratic NON-residue** of 23.