Solutions to the Attendance Quiz # 8 for Dr. Z.'s Number Theory Course for Sept. 30, 2013

1. Use the Euclidea algoithm to find $gcd(49, 140)$

Sol. to 1: Since $\mathbf{140} = 2 \cdot \mathbf{49} + 42$, we have

$$gcd(140, 49) = gcd(49, 42) \quad .$$

Since $\mathbf{49} = 1 \cdot 42 + 7$, so $r = 7$, and we have

$$gcd(49, 42) = gcd(42, 7) \quad .$$

Since $42 = 7 \cdot 6 + 0$, $r = 0$, and the gcd is 7.

Ans. to 1: $gcd(140, 49) = 7$

2. Find out whether it is possible to express 1 as a linear combination $1 = m35 + n9$ for some integers m and n, and if it is, find it.

Sol.to 2:

$\mathbf{35} = 3 \cdot \mathbf{9} + 8$ so $r = 8$ and

$$gcd(35, 9) = gcd(9, 8) \quad , \quad 8 = \mathbf{35} - 3 \cdot \mathbf{9} \quad .$$

Now $\mathbf{9} = 1 \cdot 8 + 1$ so $r = 1$ and

$$gcd(9, 8) = gcd(8, 1) \quad , 1 = \mathbf{9} - 1 \cdot 8$$

$$= \mathbf{9} - (\mathbf{35} - 3 \cdot \mathbf{9}) = 4 \cdot \mathbf{9} - \mathbf{35} \quad .$$

Now $8 = 8 \cdot 1 + 0$ so $r = 0$ and the previous r (namely 1) is the gcd, and

$$1 = 4 \cdot \mathbf{9} - \mathbf{35} \quad .$$

Ans. to 2: It is possible to express 1 as a linear combination of 9 and 35: $1 = -\mathbf{35} + 4 \cdot \mathbf{9}$. So $m = -1$ and $n = 4$.