Solutions to Attendance Quiz # 7 for Dr. Z.'s Number Theory Course for Sept. 26, 2013

1. Find $q(101, 17)$ and $r(101, 17)$.

Sol. to 1: $5 \cdot 17 = 85 < 101$, but $6 \cdot 17 = 102 \geq 101$, so $q(101, 17) = 5$ and $r(101, 17) = 101 - q(101, 17) \cdot 17 = 101 - 5 \cdot 17 = 101 - 85 = 16$.

Ans. to 1: $q(101, 17) = 5$ and $r(101, 17) = 16$.

Comment: Almost everyone got it right.

2. Use the clever way to find $Div(210)$.

Sol. to 2: Using the algorithm of the **Fundamental Theorem of Arithmetic**

$$210 = 2^1 \cdot 3^1 \cdot 5^1 \cdot 7^1$$

So

$$Div(210) = \{1, 2\} \cdot \{1, 3\} \cdot \{1, 5\} \cdot \{1, 7\}$$

$$Div(210) = (\{1, 2\} \cdot \{1, 3\}) \cdot (\{1, 5\} \cdot \{1, 7\})$$

$$= \{1 \cdot 1, 1 \cdot 3, 2 \cdot 1, 2 \cdot 3\} \cdot \{1 \cdot 1, 1 \cdot 7, 5 \cdot 1, 5 \cdot 7\}$$

$$= \{1, 2, 3, 6\} \cdot \{1, 5, 7, 35\}$$

$$= \{1 \cdot 1, 1 \cdot 5, 1 \cdot 7, 1 \cdot 35 \quad ,$$

$$2 \cdot 1, 2 \cdot 5, 2 \cdot 7, 2 \cdot 35 \quad ,$$

$$3 \cdot 1, 3 \cdot 5, 3 \cdot 7, 3 \cdot 35 \quad ,$$

$$6 \cdot 1, 6 \cdot 5, 6 \cdot 7, 6 \cdot 35\}$$

$$= \{1, 5, 7, 35 \quad ,$$

$$2, 10, 14, 70 \quad ,$$

$$3, 15, 21, 105 \quad ,$$

$$6, 30, 42, 210\} \quad .$$

This is a **correct** answer, but it is polite to list the elements in increasing order. so a

Polite Answer to 2 is:

$$\{1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210\} \quad .$$

Comment: About %60 got it right. It is a good idea to check by using the formula for the number of divisors

$$|Div(n)] = \prod_{i=1}^{k} (a_i + 1) \quad .$$

Here $k = 4$ and $a_1 = 1, a_2 = 1, a_3 = 1, a_4 = 1$, so

$$|Div(210)| = (1+1)(1+1)(1+1)(1+1) = 2^4 = 16 \quad ,$$

and indeed we got 16 elements in our answer.