Solutions to Attendance Quiz # 4 for Dr. Z.'s Number Theory Course for Sept. 16, 2013

1. Convert 24 to binary, in (i) sparse notation (ii) dense notation (iii) positional notation.

Solution to 1. The largest power of $2 \leq 24$ is 2^4, so

$$B(24) = 2^4 + B(24 - 2^4) = 2^4 + B(24 - 16) = 2^4 + B(8) \quad .$$

The largest power of $2 \leq 8$ is 2^3, so

$$B(8) = 2^3 + B(8 - 2^3) = 2^3 + B(0) = 2^3 \quad .$$

(Since $B(0)$ is empty).

Hence

(i) **sparse notation** $24 = 2^4 + 2^3$.

To get the **dense notation** we include all the powers of $2 \leq 2^4$ that do not show up and stick coefficient 0. For those that **do** show up, with stick coefficient 1

(ii) **dense notation** $24 = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$.

Finally to get the positional notation, we just write down the coefficients

$$24 = 11000 \quad (base \quad 2) \quad .$$

2. Use Karatsuba's algorithm (no credit for other methods!), to find the product

$$12 \cdot 23 \quad .$$

Solution to 2:

We need to compute

$$(1 \cdot 10 + 2) \cdot (2 \cdot 10 + 3) \quad .$$

So $a = 1, b = 2, c = 2, d = 3$.

$$a \cdot c = 1 \cdot 2 = 2$$

$$b \cdot d = 2 \cdot 3 = 6$$

$$(a + c)(b + d) = (1 + 2)(2 + 3) = 3 \cdot 5 = 15$$

$$(a + c)(b + d) - ab - bd = 15 - 2 - 6 = 7$$

Since

$$(10a + b)(10c + d) = 100ac + 10((a + b)(c + d) - ac - bd) + bd$$

We have

$$(1 \cdot 10 + 2) \cdot (2 \cdot 10 + 3) = (2 \cdot 1) \cdot 100 + 10 \cdot 7 + 6 = 276 \quad .$$

Ans. to 2.: $12 \cdot 23 = 276$.