Solutions to Attendance Quiz # 23 for Dr. Z.'s Number Theory Course for Dec. 2, 2013

1.: Use Euler's recurrence to compute $p(7)$, if someone told you that

$$p(0) = 1 \quad , \quad p(1) = 1 \quad , \quad p(2) = 2 \quad , \quad p(3) = 3 \quad , \quad p(4) = 5 \quad , p(5) = 7 \quad , \quad p(6) = 11,$$

Sol. of 1: The pentagonal numbers are $1, 2$ (for $j = -1, 1$), $5, 7$ (for $j = -2, 2$), etc. (we don't need to go further, since $n = 7$). So

$$p(n) = p(n-1) + p(n-2) - p(n-5) - p(n-7) + \ldots,$$

and with $n = 7$:

$$p(7) = p(7-1) + p(7-2) - p(7-5) - p(7-7) = p(6) + p(5) - p(2) - p(0) = 11 + 7 - 2 - 1 = 15 \quad .$$

Ans. to 1: $p(7) = 15$.

2. Apply the Bressoud-Zeilberger mapping ϕ to the following λ. Then compute $\phi^2(\lambda)$ and make sure that you get λ back.

$$j = 0 \quad , \quad \lambda = (4, 2, 1, 1) \quad ,$$

Sol. to 1: $\lambda_1 = 4, t = 4, j = 0$, so $t + 3j = 4$. Since $4 \geq 4$, The first case applies, so we stick a new part of size $t + 3j - 1 = 3$ at the front, and subtract 1 from the remaining parts, getting

$$\phi(\lambda) = (3, 3, 1, 0, 0) = (3, 3, 1) \quad ; \quad j = -1.$$

(Of course, we discard the 0's.)

Now let's apply ϕ to $(3, 3, 1)$. **Now** $\lambda_1 = 3$, $t = 3$, and $j = -1$. So $t + 3j = 3 + 3(-1) = 0$. Since $0 < 3$, the bottom case applies, and, we delete λ_1, add 1 to the remaining parts, and add $\lambda_1 - 3j - t - 1 = 3 - 3(-1) - 3 - 1 = 2$ additional 1-s at the end, getting

$$\phi(3, 3, 1) = (3 + 1, 1 + 1, 1, 1) = (4, 2, 1, 1) \quad ; \quad j = 0 \quad .$$

Yea! We got λ back.