Solutions to Attendance Quiz # 19 for Dr. Z.'s Number Theory Course for Nov. 11, 2013

1. Compute **a.** $\mu(100)$ **b.** $\mu(1001)$

Sol. to 1:

a. $100 = 2^2 \cdot 5^2$. Since at least one of the powers is larger than one, (in this case both are) $\mu(100) = 0$.

b. $1001 = 7 \cdot 11 \cdot 13$, so $\mu(1001) = (-1)^3 = -1$.

2. Check empirically that $\sum_{d|30} \mu(d) = 0$.

Sol. to 2: $30 = 2 \cdot 3 \cdot 5$, so

$$Div(30) = \{1,2\}\cdot\{1,3\}\cdot\{1,5\} = \{1,2,3,6\}\cdot\{1,5\} = \{1,2,3,6\}\cup\{5,10,15,30\} = \{1,2,3,5,6,10,15,30\}$$

Now

$$\mu(1) = 1 \quad , \quad \mu(2) = -1 \quad , \quad \mu(3) = -1 \quad , \quad \mu(5) = -1 \quad ,$$

$$\mu(6) = \mu(2\cdot3) = (-1)^2 = 1 \quad , \quad \mu(10) = \mu(2\cdot5) = (-1)^2 = 1 \quad , \quad \mu(15) = \mu(3\cdot5) = (-1)^2 = 1 \quad ,$$

$$\mu(30) = \mu(2\cdot3\cdot5) = (-1)^3 = -1 \quad .$$

So

$$\sum_{d|30} \mu(d) = \mu(1)+\mu(2)+\mu(3)+\mu(5)+\mu(6)+\mu(10)+\mu(15)+\mu(30) = 1-1-1-1+1+1+1-1 = 0 \quad .$$

Yea!