Solutions to the Attendance Quiz # 15 for Dr. Z.'s Number Theory Course for Oct. 28, 2013

1. Using the definition, find $\phi(12)$.

Sol. of 1:

$$gcd(1,12)=1 \quad , \quad gcd(2,12)=2 \quad , \quad gcd(3,12)=3 \quad , \quad gcd(4,12)=4 \quad , \quad gcd(5,12)=1 \quad ,$$

$$gcd(6,12)=6 \quad , \quad gcd(7,12)=1 \quad , \quad gcd(8,12)=4 \quad , \quad gcd(9,12)=3 \quad , \quad gcd(10,12)=2 \quad ,$$

$$gcd(11,12)=1 \quad , \quad gcd(12,12)=12 \quad .$$

The set of integers relatively prime to 12 is: $\{1,5,7,11\}$. Since its number of elements is 4, we have:

Ans. to 1: $\phi(12)=4$.

2. Using the formula, find $\phi(1500)$.

Sol. to 2: We first do a **prime-power factorization**:

$$1500 = 15 \cdot 100 = 3 \cdot 5 \cdot (2 \cdot 5)^2 = 2^2 \cdot 3 \cdot 5^3 \quad .$$

So

$$\begin{aligned} \phi(1500) &= (2^2 \cdot 3 \cdot 5^3) \cdot (1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5}) \\ &= (2^2 \cdot 3 \cdot 5^3) \cdot (\frac{1}{2})(\frac{2}{3})(\frac{4}{5}) \\ &= (2^1 \cdot 3^0 \cdot 5^2) \cdot (1 \cdot 2 \cdot 4) \\ &= (50)(8) = 400 \quad . \end{aligned}$$

Ans. to 2: $\phi(1500)=400$.