Solutions to Attendance Quiz # 14 for Dr. Z.'s Number Theory Course for Oct. 24, 2013

1. Use Korslet's theorem to decide whether or not the following are Carmichael numbers 1728.

Sol. to 1.: $1728 = 2 \cdot 864 = 2^2 \cdot 432$. This means that 1728 is **not** square-free (it contains at least one prime (2) to a power higher than 1. So the very first condition to being a Carmichael number is wrong, so there is no way!

Ans. to 1: 1728 is **not** a Carmichael number.

2. Use the Fermat primality test to investigate whether the following integers are composite or probably primes

a. 11 **b.** 10

Sol. to 2a: Let's pick $a = 2$, and compute $2^{10} \pmod{11}$.

$$2^1 \equiv 2 \pmod{11}$$

$$2^2 \equiv 2^2 \pmod{11} \equiv 4 \pmod{11}$$

$$2^4 \equiv 4^2 \pmod{11} \equiv 5 \pmod{11}$$

$$2^8 \equiv 5^2 \pmod{11} \equiv 3 \pmod{11}$$

So

$$2^{10} = 2^8 \cdot 2^2 = 3 \cdot 4 \equiv 1 \pmod{11} \quad .$$

Hence 11 is **probably a prime**. If you want to be more sure, then try other a's.

Note: Of course, we know already that 11 is prime for sure. This is just to familiarize ourselves with the method. In real life computers apply it to very large integers.

Sol. to 2b: Let's pick $a = 2$, and compute $2^0 \pmod{10}$.

$$2^1 \equiv 2 \pmod{10}$$

$$2^2 \equiv 2^2 \pmod{10} \equiv 4 \pmod{10}$$

$$2^4 \equiv 4^2 \pmod{10} \equiv 6 \pmod{10}$$

$$2^8 \equiv 6^2 \pmod{10} \equiv 6 \pmod{11}$$

So

$$2^{10} = 2^8 \cdot 2^2 = 6 \cdot 4 \equiv 4 \pmod{10} \quad .$$

Since this is **not** 1, we know **for sure** that $n = 10$ is composite.