Solutions to Attendance Quiz # 12 for Dr. Z.'s Number Theory Course for Oct. 17, 2013

1. Using divisiblity tests, determine which of the following integers (written in the usual, base 10, way) is divisible by (i) 9 (ii)11 (iii)7.

a. 683444

b. 17888

c. 139135

Sol. of 1:

a

(i) $6+8+3+4+4+4 \pmod 9 = 2$ so it **not** divisible by 9

(ii) $6-8+3-4+4-4 \pmod{11} = 8$ so it is **not** divisible by 11

(iii) $684-444 \pmod 7 = 240 \pmod 7 = 2$ so it is **not** divisible by 7

b.

(i) $1+7+8+8+8 \pmod 9 = 5$ so it **not** divisible by 9

(ii) $1-7+8-8+8 \pmod{11} = 2$ so it is **not** divisible by 11

(iii) $888-17 \pmod 7 = 871 \pmod 7 = 3$ so it is **not** divisible by 7

c

(i) $1+3+9+1+3+5 \pmod 9 = 4$ so it **not** divisible by 9

(ii) $1-3+9-1+3-5 \pmod{11} = 4$ so it is **not** divisible by 11

(iii) $139-135 \pmod 7 = 4 \pmod 7$ so it is **not** divisible by 7

2. Using the Perpetual calendar algorithm, find out on what day of the week is going to be Oct. 17, 5000.

Hint: Find out how many leap years will be from now until 5000.

Sol. of 2.:

(i) The number of years until Oct. 17, 2013 is $5000-2013 = 2987$

(ii) The number of leap years, ignoring the excepctions and the exceptions to the exceptions is

$\lfloor (5000-2012)/4 \rfloor = 747$,

(iii) The number of multiples of 100, starting at 2013 until 5000 (including 5000) is $50 - 20 = 30$

(iv) The number of multiples of 400 until 5000, starting with 2400 is $\lfloor (5000-2000)/400 \rfloor = \lfloor 30/4 \rfloor = 7$.

So the total number of leap years (extra days) is:

$$747 - 30 + 7$$

And the total number of days that elapsed, mod 7 is

$$2987 + 747 - 30 + 7 \pmod 7 = 5 + 5 - 2 + 0 \pmod 7 = 1$$

But **today**, Oct. 17, 2013, is **Thursday**, i.e. Day 5. So Oct. 17, 5000 is going to be Day $5+1=6$, in other words, a Friday.

Ans. to 2: Oct. 17, 5000 will fall on a Friday.