NAME: (print!) ____

E-Mail address: ____

MATH 356, Dr. Z. , Exam I, Mon., Oct. 14, 2024 8:30-9:50am, SEC-204

No Calculators! No Cheatsheets!
Write the final answer to each problem in the space provided. Incorrect answers (even due to minor errors) can receive at most one half partial credit, so please check and double-check your answers.

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

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tot.: (out of 100)

1. (10 pts.) Solve, if possible, the following linear congruences

$$3x \equiv 8 \pmod{35}$$

Ans.: $x =$

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2. (10 pts.) Find the two last (decimal) digits of 1013^{17}

Ans.:

3. (10 pts.) Find out whether it is possible to express 1 as a linear combination

$$1 = m \cdot 44 + n \cdot 91 \quad ,$$

for some integers m and n, and if it is possible, find it.

Ans.: $m =$ $\qquad n =$ $\qquad$.

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4. (10 pts.) Find the set of divisors of 105

Ans.: $Div(105) = \{ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \}$

5. (10 pts.) Express 25025 as a product of prime powers.

Ans.:

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6. (10 pts.)
Recall that the *Euclid-Mullin sequence* is defined by $M_1 = 2$, and for $n > 1$, M_n is the smallest prime divisor of $M_1 \cdot M_2 \cdots M_{n-1} + 1$.
Find the first **five** terms of the Euclid-Mullin sequence.

Ans.: $M_1 = 2$, $M_2 =$, $M_3 =$, $M_4 =$, $M_5 =$.

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7. (10 pts.) Express the integer 301 (writen in our usual (base 10) notation) in base 5, in (i) sparse notation (4 pts) (ii) dense notation (3 pts) (iii) base-five positional notation (3 pts)

Ans.: (i)

(ii)

(iii)

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8. (10 pts.) Use Karatsuba's algorithm (no credit for other methods!), to find the product

$$99 \cdot 53 \quad .$$

Reminder: $(10a + b)(10c + d) = 100ac + 10((a + b)(c + d) - ac - bd)) + bd$.

Ans.:

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9. (10 pts.) Let F_n be the Fibonacci numbers. Give a Zeilberger-style proof of the following identity, using $N_0 = 3$, of the identity

$$\sum_{i=1}^{2n-1} F_i F_{i+1} = F_{2n}^2 \quad .$$

Disclaimer: To get a fullly rigorous proof, we need $N_0 = 5$, but I am being nice, so I only ask you to do it for $n = 1, 2, 3$, so this is a semi-rigorous proof.

10. (10 pts.)
Use the Fundamental Theorem of Discrete Calculus to prove

$$\sum_{i=1}^{n} 3^i = \frac{3}{2}(3^n - 1) \quad .$$