Dr. Z.'s Number Theory Homework assignment 1

1. Using *unary* (no credit for other methods!), compute

a. 11 + 1111111

b. 111111 + 11

2. Using *unary* (no credit for other methods!), compute

a. 11 × 111

b. 1111 × 1111111

3. Write the integers 0 through 9 in von-Neumann notation

4. List some **natural** members of Frege's class representing five

5. Write down (**without peeking**) Peano's axioms for the Natural Numtbers .

Dr. Z.'s Number Theory Homework assignment 2

Version of Sept. 12, 2013.

1. For the following (i) Guess a nice formula by inspection (ii) Give a rigorous proof of your guessed formula, using the Fundamental Theorem of Discrete Calculus (iii) Give a fully rigorous Zeilberger-style proof.

Added Sept. 12, 2013: It turns out that to guess the right formula, you need an IQ of 170 or up, so you could cheat and first figure out the answer using algebra and the famous formulas for $\sum_{i=1}^n i$, $\sum_{i=1}^n i^2$, and $\sum_{i=1}^n i^3$ (that you can look up). Another way is by *undetermined coefficients*, since you know the degree of the right side (it is one more than the degree of the summand) and since $S(0) = 0$ you can write $S(n) = a_1 n + \ldots + a_d n^d$ (if d is the degree), and plug-in $n = 1, 2, \ldots, d$ and use linear algebra to solve for $a_1, \ldots, a_d$.

a.

$$\sum_{i=1}^{n} 3i - 1 \quad ,$$

b.

$$\sum_{i=1}^{n} (2i-1)^2 \quad ,$$

c.

$$\sum_{i=1}^{n} (2i-1)^3 \quad .$$

2. Prove the following identities (i) using the Fundamental Theorem of Discrete Calculus (ii) using a Zeilberger-style proof via checking special cases.

a.

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 \quad .$$

b.

$$\sum_{i=1}^{n} i(i+1)(i+2) = \frac{1}{4} n(n+1)(n+2)(n+3) \quad .$$

3. Use the Fundamental Theorem of Discrete Calculus to prove

a.

$$\sum_{i=1}^{n} q^i = \frac{q(1-q^n)}{1-q} \quad ,$$

for any number q, and any positive integer n .

b.

$$\sum_{i=1}^{n} i q^i = \frac{n q^{n+2} - n q^{n+1} - q^{n+1} + q}{(q-1)^2}$$

for any number q, and any positive integer n .

4. In a small Italian village there are 100 married couples, except for the priest (so there are 100 married men and 100 married women, and one priest). One Sunday, the priest announces in Church:

It came to my knowledge that **at least** one woman is unfaithful. It is a great sin to shoot your cheating wife unless you are **completely** sure that she is cheating on you. On the other hand, if you are sure, by all means, you should shoot her exactly at **midnight** (after you found out for sure).

Of course, due to gossip, everyone knows about everybody else's wife, **but not** about one's own wife.

For the first 99 midnights, nothing happened, but at the 100th midnight, there were heard 100 shots, meaning that all the husbands knew for sure that their wives were cheating on them. Use mathematical induction to explain this.

Dr. Z.'s Number Theory Homework assignment 3

1. Write down F_n for $1 \leq n \leq 10$.

2. List all the 1-2 walks from 0 to 7. Count them and make sure that you get $f_7 = F_8$.

3. Prove the following identities (using the fact that F_n is a linear combination of α^n and β^n where $\alpha, \beta = \frac{1 \pm \sqrt{5}}{2}$.

a.

$$F_{n+3} - F_n = 2F_{n+1} \quad .$$

b.

$$F_{n-2} + F_{n+2} = 3F_n \quad .$$

c.

$$F_{n+4} - F_{n+3} + 2F_{n+2} - 3F_{n+1} - 3F_n = 0 \quad .$$

4. Use the combinatorial model (in terms of paths) to prove that for every positive integer n,

$$F_{m+n+1} = F_{m+1}F_{n+1} + F_m F_n \quad .$$

5. Give a Zeilberger-style proof of the following identities, using the provided N_0

a.

$$F_{n+1}F_n - F_{n-1}F_{n-2} = F_{2n-1} \quad , \quad N_0 = 5 \quad .$$

b.

$$\sum_{i=1}^{2n-1} F_i F_{i+1} = F_{2n}^2 \quad , \quad N_0 = 5 \quad .$$

Dr. Z.'s Number Theory Homework assignment 4

1. Convert the following to binary, in (i) sparse notation (ii) dense notation (iii) positional notation.

a. 17 **b.** 257 **c.** 515 .

2. Convert the following to base 5, in (i) sparse notation (ii) dense notation (iii) positional notation.

a. 130 **b.** 631

3. Find, in base 6,

$$(5 \cdot 6^3 + 1 \cdot 6^0) + (4 \cdot 6^3 + 5 \cdot 6^1 + 5 \cdot 6^0)$$

4. Find, in base 6,

$$(5 \cdot 6^3 + 1 \cdot 6^0) \cdot (4 \cdot 6^3 + 5 \cdot 6^1 + 5 \cdot 6^0) \quad .$$

5. Use Karatsuba's algorithm (no credit for other methods!), to find the product

$$87 \cdot 93 \quad .$$

Dr. Z.'s Number Theory Homework assignment 5

1. Decide whether the following integers are prime or composite, using (i) the very stupid way (ii) the stupid way (iii) the OK way

a. 99 **b.** 37

2. Decide whether the following integers are prime or composite, using the OK way

a. 101 **b.** 1001 .

3. Use the sieve of Eratosthenes to list **all** the prime numbers ≤ 140.

4. Prove that there are infinitely many primes.

5. Find, by only using paper-and-pencil the first six terms of the Euclid-Mullin sequence.

6. (Optional, extra credit) A twin-prime-pair is a pair of two prime numbers that differ by 2. The first few are $(3,5)$, $(5,7)$, $(11,13)$. Prove that there are infitely many twin-prime pairs.

Dr. Z.'s Number Theory Homework assignment 6

1.: Using the recursive algorithm **directly**, find the proudct-of-prime-powers representation of

a. 1000 **b.** 7007 **c.** 2700 **d.** 1024

2. Use any method to find the proudct-of-prime-powers representation of

a. a googol : 10^{100} **b.**: a googolplex $10^{10^{100}}$

c.

$$(3^3 \cdot 5^{101} \cdot 11^{11}) \cdot (3 \cdot 5^3 \cdot 11^{12} \cdot 13^2 \cdot 41^2) \quad .$$

3. (challenge) Yesterday I went to a bookstore and I overheard a conversation between two people

A. This is a nice book, are you going to buy it?

B. I like it, since the number in the title is the product of the ages of my three children, can you figure their ages?

A. Of course not, there are several possibilities.

B. But let me add that all the ages are distinct (i.e. I have neither twins nor a triplet), and their ages is at least 2.

A. Yes, now I know the ages of your children.

Can **YOU** figure out their ages?

Dr. Z.'s Number Theory Homework assignment 7

1. Find $q(a,b)$ and $r(a,b)$ for the following

i. $a=6$, $b=4$ **ii.** $a=61$, $b=17$ **iii.** $a=120$, $b=30$ **iv.** $a=120$, $b=121$.

2. Use the stupid way to find **i.** $Div(16)$, **ii.** $Div(13)$,

3. Use the clever way to find $Div(1001)$.

4. Use the VERY stupid way to find **i.** $gcd(15,35)$ **ii.** $gcd(100,30)$

5. Find the following, using the neither-stupid-nor-clever way.

i. $gcd(15,55)$ **ii.** $gcd(1001,154)$ **iii.** $gcd(1000,1500)$.

Dr. Z.'s Number Theory Homework assignment 8

1. Use the Euclidea algoithm to find

i. $gcd(77,143)$ **ii.** $gcd(190,46)$ **iii.** $gcd(1001,500)$

2. Find

i. $gcd(10,20,70,90)$ **ii.** $gcd(6,9,25)$ **iii.** $gcd(35,49,77)$

3. Find out whether it is possible to express 1 as a linear combination $1=ma+nb$ for some integers m and n, and if it is, find it.

i. $a=55$, $b=8$ **ii.** $a=100$, $b=55$ **iii.** $a=1001$, $b=85$ **iv.** $a=21$, $b=13$.

Dr. Z.'s Number Theory Homework assignment 9

1. Find

i. $1001 \pmod{13}$ **ii.** $1001 \pmod{17}$ **iii.** $10001 \pmod{1007}$

2. True or False?

i. i. $10^{100}+10^{98}+1 \equiv 10^{98}+1 \pmod{10^{100}}$ **ii.** $10^{100}+10^{98}+1 \equiv 10^{98}+2 \pmod{10^{100}}$

3. Find $91+87+85+103 \pmod{105}$

(**Tip**: If $a>m/2$ then it is more convenient to replace a by $a-m$.

4. i. Compute $2001 \cdot 207 \pmod{1000}$.

ii. Compute $2009 \cdot 2011 \pmod{2010}$.

5. Compute $13^5 \pmod{15}$ using (a) the slow way. (b) the fast way. (c) the other fast way.

6. Compute $13^{18} \pmod{15}$ using (a) the fast way. (b) the other fast way.

7. Find the last (decimal) digits of 7^{1024}

8. Find the three last (decimal) digit of 7^{1024}

Dr. Z.'s Number Theory Homework assignment 10

1. Without actually solving, find out how many solutions there are in $\{0, 1, \ldots n-1\}$ where n is the modulo.

i. $7x \equiv 2 \pmod{11}$

ii. $42x \equiv 12 \pmod{66}$

iii. $15x \equiv 64 \pmod{35}$

iv. $10^{10}x \equiv 64 \pmod{11^{100}}$

2. Using Brute Force, solve

$$5x \equiv 6 \pmod{13} \quad .$$

3.: Find, or explain why it does not exist

i. $3^{-1} \pmod{35}$.

ii. $7^{-1} \pmod{35}$.

iii. $7^{-1} \pmod{125}$.

iv. $5^{-1} \pmod{21}$.

4. Solve, if possible, the following linear congruences.

i. $3x \equiv 8 \pmod{35}$

ii. $7x \equiv 100 \pmod{35}$

iii. $7x \equiv 97 \pmod{125}$.

iv. $25x \equiv 75 \pmod{105}$.

5. Solve the system of linear congruences

$$3x \equiv 8 \pmod{35} \quad , \quad 5x \equiv 2 \pmod{11} \quad .$$

Dr. Z.'s Number Theory Homework assignment 11

1. Using the first way, find the unique x between 0 and 34 such that

a.

$$x \equiv 4 \pmod 5 \quad , \quad x \equiv 2 \pmod 7 \quad .$$

b.

$$x \equiv 1 \pmod 5 \quad , \quad x \equiv 6 \pmod 7 \quad .$$

c.

$$x \equiv 2 \pmod 5 \quad , \quad x \equiv 5 \pmod 7 \quad .$$

2. Using the second way (the formula) find the unique x between 0 and 34 such that

a.

$$x \equiv 4 \pmod 5 \quad , \quad x \equiv 2 \pmod 7 \quad .$$

b.

$$x \equiv 1 \pmod 5 \quad , \quad x \equiv 6 \pmod 7 \quad .$$

c.

$$x \equiv 2 \pmod 5 \quad , \quad x \equiv 5 \pmod 7 \quad .$$

3. Using the make-a-table way, find the unique x between 0 and 59 such that

a.

$$x \equiv 3 \pmod 4 \quad , \quad x \equiv 2 \pmod 3 \quad , \quad x \equiv 3 \pmod 5 \quad .$$

b.

$$x \equiv 2 \pmod 4 \quad , \quad x \equiv 1 \pmod 3 \quad , \quad x \equiv 4 \pmod 5 \quad .$$

c.

$$x \equiv 3 \pmod 4 \quad , \quad x \equiv 0 \pmod 3 \quad , \quad x \equiv 1 \pmod 5 \quad .$$

4. Using the formula, find the unique x between 0 and 2001 such that

$$x \equiv 1 \pmod 2 \quad , \quad x \equiv 5 \pmod 7 \quad , \quad x \equiv 6 \pmod{11} \quad , \quad x \equiv 4 \pmod{13} \quad .$$

5. Use any method to find the smallest non-negative integer x such that

$$x \equiv 3 \pmod{1024} \quad , \quad x \equiv 3 \pmod{121} \quad , \quad x \equiv 3 \pmod{169} \quad , \quad x \equiv 3 \pmod{17} \quad , \quad x \equiv 3 \pmod{529} \quad .$$

Dr. Z.'s Number Theory Homework assignment 12

1. Using divisiblity tests, determine which of the following integers (written in the usual, base 10, way) is divisible by (i) 9 (ii)11 (iii)7.

a. 6844430439

b. 178888708386

c. 1391356620

2. Using divisiblity tests determine which of the following integers are divisible by 99 (i.e. by **both** 9 and 11)

a. 976865076

b. 976865076

c. 171928253381

3. Using divisiblity tests determine which of the following integers, written in base 7 are divisible (i) by 6 (ii) 11 (base 7) (alias 8 base 10)

a. 316653313

b. 35145

c. 610040033223

4. Using the Perpetual calendar algorithm, find out on what day of the week

(i) were you born?

(ii) you will turn 60-years-old

(iii) turn 200-years-old

5. Using the Perpetual calbigskipar algorithm, find out on what day of the week

(i) was you mother born?

(ii) was your father born?

(iii) for each brother and sister find out the day of the week that they were born.

Dr. Z.'s Number Theory Homework assignment 13

1. Check empirically Wilson's theorem for (a) $n = 3$ (b) $n = 11$ (c) $n = 23$

(Hint: do not compute 22!, use modular arithmetic to find $22! \pmod{23}$)

2. Illustrate the proof of Wilson's theorem for $p = 17$.

3. Check, empirically, Fermat's little theorem for $p = 31$, and $a = 5$.

(Hint: do not compute 2^{23} and then divide by 23 to get the remainder! Use modular arithmetic modulo 31)

4. Illustrate the Second proof of Fermat's little theorem with $p = 7$ and $a = 2$.

5. How many necklaces of length 5 are there with green, and blue beads? Write them all down.

6. How many necklaces of length 101 are there with 10 colors?

7. Read, understand, and be able to reproduce (in a quiz or exam) the proof of Wilson's Theorem.

8. Read, understand, and be able to reproduce the three proofs of Fermat's little theorem.

Dr. Z.'s Number Theory Homework assignment 14

1. Use Korslet's theorem to decide whether or not the following are Carmichael numbers

a. 1729 **b.** 2000 **c.** 105 **d.** 6601.

2. Use the Fermat primality test to investigate wheter the following integers are composite or probably primes (by picking up-to two witnesses)

a. 15 **b.** 9 **c.** 11 **d.** 23.

3. Use the Miller-Rabin primality test to investigage whether the following integers n are prime or composite by picking two (if necessary) random a's between 2 and $n - 2$

a. 30 **b.** 13 **c.** 14 **d.** 11.

Dr. Z.'s Number Theory Homework assignment 15

1. Using the definition, find

a. $\phi(16)$, **b.** $\phi(12)$.

2. Using the formula, find

a. $\phi(1001)$, **b.** $\phi(1000)$, **c.** $\phi(10^{100})$ (leave it factored) .

3. Verify Euler's Classical Formula for

a. $n = 14$, **b.** $n = 16$.

4. State and prove the multiplicative property for $\phi(n)$

5. State and prove the formula for Euler's Totient Function.

6. State and prove Euler's Classical Formula for the sum-over-divisors of n of ϕ.

Dr. Z.'s Number Theory Homework assignment 16

Version of Oct. 31, 2013 [PLEASE DISREGARD EARLIER VERSION]

1. Check Euler's theorem for (**a.**) $n = 15$ (**b.**) $n = 24$ (**c.**) $n = 21$

2. For the following primes p and q (let $n = pq$) public key e, and encrypted message c

(i) Check that e is an OK key, i.e. that it is coprime to $\phi(n)$.

(ii) Find the deciphering key, d, such that $de \equiv 1 \pmod{\phi(n)}$

(iii) Suppose Alice sent you the encrypted message c. Check that this is an OK message (coprime to n), and if it is find her original message?, m

a. $p = 11$, $q = 7$, $e = 7$, $c = 20$

b. $p = 11$, $q = 5$, $e = 9$, $c = 19$

c. $p = 3$, $q = 13$, $e = 7$, $c = 16$

d. $p = 7$, $q = 17$, $e = 5$, $c = 11$

e. $p = 7$, $q = 17$, $e = 3$, $c = 11$

f. $p = 7$, $q = 17$, $e = 5$, $c = 17$

3. State and prove Euler's Theorem.

Dr. Z.'s Number Theory Homework assignment 17

1. Spell-out the explanation for the important formula for $\sigma(n)$ for $n = 50$.

2. Spell-out the explanation for the important formula for $\sigma_3(n)$ for $n = 24$.

3. Verify the formulas for $d(n)$, $\sigma_1(n)$, $\sigma_2(n)$ and $\sigma_3(n)$ for for **a.** $n = 10$ **b.** $n = 26$ **c.** $n = 16$

4. Verify the Dirichlet series for $\sigma(n)$ for up to $n = 10$

5. Verify the Dirichlet series for $\sigma_3(n)$ for up to $n = 5$

6.: State the Lambert series for $\sigma_3(n)$ and verify it through $n = 8$

7. (challenge) Using a computer (Maple or otherwise), go towards proving the Riemann Hypothesis by checking the assertion in Robin's Theorem as far as you can.

Dr. Z.'s Number Theory Homework assignment 18

1. Which of the following are perfect numbers? Explain!

a. 496 ; **b.** 100 ; **c.** 1000 ; **d.** 8128.

2. Using the Lucas-Lehmer test (no credit for other methods), show that $M_{11} = 2^{11} - 1 = 31$ is **not** a Mersenne prime. **Note**: You may use a calculator (or computer) to compute S_9.

3. (Without peeking at your notes), prove that if p is a prime, and $2^p - 1$ is also a prime, then

$$2^{p-1} \cdot (2^p - 1)$$

is a perfect number.

4. (Without peeking at your notes), prove that if p and q are distinct odd primes,then pq can **not** be a perfect number.

5. (Without peeking at your notes), state precisely the Lucas-Lehmer test for testing whether M_p is a Mersenne prime.

6. (Without peeking at your notes), prove that if n is an integer that is not a prime, then $2^n - 1$ is not a prime either.

Dr. Z.'s Number Theory Homework assignment 19

1. Compute $\mu(n)$ for $1 \leq n \leq 20$.

2. Check empirically that $\sum_{d|105} \mu(d) = 0$.

3. State the Möbius inversion formula, and check it empirically for all $n \in Div(42)$

4. (Without peeking) prove that for all n, $\sum_{d|n} \mu(n) = 0$.

5. Express the Dirichlet series for $\mu(n)$ in terms of the Riemann Zeta function and prove it.

6. (Challenge) Prove that for every $\epsilon > 0$, you can find a constant C_ϵ such that

$$\sum_{i=1}^{n} \mu(i) \leq C_\epsilon n^{\frac{1}{2}+\epsilon} \quad .$$

Dr. Z.'s Number Theory Homework assignment 20

1. By 'brute force', Find the set of quadratic residues, and the set of quadratic non-residues of the following primes.

a. $p = 19$ **b.** $p = 23$ **c.** $p = 31$

2. Using the important test (no credit for brute force!) find

a. $\left(\frac{7}{19}\right)$ **b.** $\left(\frac{17}{23}\right)$ **c.** $\left(\frac{3}{29}\right)$

3. Using the Quadratic Reciprocity Law and Rules 1-3, find (no credit for other methods)

a.

$$\left(\frac{7}{11}\right)$$

b.

$$\left(\frac{41}{61}\right)$$

c.

$$\left(\frac{19}{97}\right)$$

Dr. Z.'s Number Theory Homework assignment 21

Version of Nov. 17, 2013 (please discard previous versions).

1. Using $\mathcal{P}(n,k) = \sum_{r=1}^{k} k\mathcal{P}(n-k,r)$, construct, systematically, $\mathcal{P}(n)$ for $1 \leq n \leq 5$.

2. For each of the following partitions λ, (i) Draw the Ferrers graph (ii) Find the conjugate partition λ'

a. $\lambda = (5, 4, 3, 2, 1)$

b. $\lambda = (9, 4, 3, 2, 1, 1, 1, 1)$

c. $\lambda = (11, 7, 6, 1)$

3. Write the following partitions in exponent notation. First the short version, then the long version (with zero exponents).

a. $(9, 5, 3, 1)$

b. $(9, 9, 9, 4, 4, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1)$

c. $(8, 8, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1)$

4. a. A partition $(\lambda_1, \ldots, \lambda_k)$ is called **distinct** if it has no repeats (i.e. every part shows up at most part), in other words

$$\lambda_1 > \lambda_2 > \ldots > \lambda_k > 0 \quad .$$

Let $dis(n)$ be the number of distinct partitions of n. Use an argument similar to the proof of Euler's theorem about the generating function of $p(n)$ to show that

$$\sum_{n=0} dis(n)q^n = \prod_{i=1}^{\infty}(1+q^i) \quad .$$

b. Use the above generating function to compute $dis(n)$ or $n = 1, 2, 3, 4, 5, 6$.

5. a. A partition $(\lambda_1, \ldots, \lambda_k)$ is called **odd** if it has no even parts, i.e. all its parts are odd. Let $odd(n)$ be the number of odd partitions of n. Use an argument similar to the proof of Euler's theorem about the generating function of $p(n)$ to show that

$$\sum_{n=0} odd(n)q^n = \prod_{i=1}^{\infty}\frac{1}{1-q^{2i+1}} \quad .$$

b. Use the above generating function to compute $odd(n)$ or $n = 1, 2, 3, 4, 5, 6$.

6. Let $q(n)$ be the number of distinct partitions of n, and $q(n, k)$ be the number of distinct partitions of n whose largest part is k.

(i) Explain why

$$q(n,k) = \sum_{r=1}^{k-1} q(n-k, r) \quad .$$

(ii) Use the above recurrence, and $q(n) = \sum_{k=1}^{n} q(n, k)$ to compute $q(n)$ for $1 \leq n \leq 6$.

Dr. Z.'s Number Theory Homework assignment 22

1. Write down the generating function for the sequence, let's call it $a(n)$, for the number of partitions of n where each part shows up at most two times and whose largest part is ≤ 4. Use it to find $a(i)$ for all $1 \leq i \leq 6$.

2. Write down the generating function for the sequence, let's call it $a(n)$, for the number of partitions of n that are congruent to 2 or 3 modulo 5. Use it to find $a(i)$ for $1 \leq i \leq 7$.

3. Write down the generating function for the sequence, let's call it $a(n)$, for the number of partitions of n that are congruent to 1 or 5 modulo 6. Use it to find $a(i)$ for $1 \leq i \leq 8$.

4. Understand and be able to replicate, under test condition Euler's proof that for every positive integer n, the number of partitions of n into distinct parts equals the number of partitions of n into odd parts.

5

(a) Write down the sets $Odd(8)$ and $Dis(n)$.

(b) For each member of $Odd(8)$ apply Glashier's bijection in the odd $\rightarrow$ distinct direction, and convince yourself that you got all the members of $Dis(8)$.

(c) For each member of $Dis(8)$ apply Glashier's bijection in the distinct $\rightarrow$ odd direction, and convince yourself that you got all the members of $Odd(8)$.

6.

i. Apply Glashier's bijection (in the odd $\rightarrow$ distinct direction) to the odd partition $(5,5,5,5,5,3,3,1,1,1)$ to get a distinct partition, call it λ

ii. Apply Glashier's bijection (in the distinct $\rightarrow$ odd direction) to the partition λ and show that you get $(5,5,5,5,5,3,3,1,1,1)$ back, as you should.

7.

i. Apply Glashier's bijection (in the distinct $\rightarrow$ odd direction) to the distinct partition $(5,4,3,2,1)$ to get an partition, call it μ

ii. Apply Glashier's bijection (in the odd $\rightarrow$ distinct direction) to the partition μ and show that you get $(5,4,3,2,1)$ back, as you should.

Dr. Z.'s Number Theory Homework assignment 23

1.: Use Euler's recurrence to compute $p(12)$, $p(13)$, and $p(14)$, if someone told you that

$$p(0)=1 \quad , \quad p(1)=1 \quad , \quad p(2)=2 \quad , \quad p(3)=3 \quad , \quad p(4)=5 \quad ,p(5)=7 \quad , \quad p(6)=11,$$

$$p(7)=15 \quad , \quad p(8)=22 \quad , \quad p(9)=30 \quad , \quad p(10)=42 \quad , \quad p(11)=56 \quad .$$

2. Apply the Bressoud-Zeilberger mapping ϕ to the following λ. Then compute $\phi^2(\lambda)$ and make sure that you get λ back.

i.

$$j=0 \quad , \quad \lambda=(5,4,3,2,1) \quad ,$$

ii.

$$j=2 \quad , \quad \lambda=(15,13,3,2) \quad ,$$

iii.

$$j=1 \quad , \quad \lambda=(5,3,3,2,2,1) \quad ,$$

3. Apply the Franklin mapping F (if applicable) to the following λ. Then compute $F^2(\lambda)$ and make sure that you get λ back.

i. $\lambda = (9, 8, 7, 6, 3)$

ii. $\lambda = (11, 10, 9, 8, 6, 3)$

iii. $\lambda = (11, 10, 9, 7, 6)$

Dr. Z.'s Number Theory Homework assignment 24

1. Evaluate the general continued fractions

a.

$$3 + \frac{2}{6 + \frac{2}{3}} \quad .$$

b.

$$2 + \frac{3}{1 + \frac{5}{2+\frac{4}{5}}} \quad .$$

2. Convert the following rational numbers into simple continued fractions.

a. $\frac{6}{17}$ **b.** $\frac{50}{19}$ **c.** $\frac{100}{13}$

3. Express as a quadractic irrationality the following infinite continued fraction.

a.

$$x = [1, 4, 1, 4, 1, 4, 1, 4, \dots \quad] \quad ,$$

where $1, 4$ repeat for ever.

b.

$$x = [2, 3, 4, 2, 3, 4, 2, 3, 4, \dots \quad] \quad ,$$

where $2, 3, 4$ repeat for ever.

4. Find a representation in the form $a + b\sqrt{Q}$ for rational numbers a and b and positive integer Q, for the following infinite, ultimately periodic, continued fractions x.

(Hint: you should use what you got in problem 3.)

a.

$$x = [5, 1, 4, 1, 4, 1, 4, 1, 4, \dots \quad] \quad ,$$

where $1, 4$ repeat for ever.

b.

$$x = [5, 1, 2, 3, 4, 2, 3, 4, 2, 3, 4, \dots \quad] \quad ,$$

where $2, 3, 4$ repeat for ever.

5. a. Convert $\sqrt{5}$ into an ultimately periodic continued fraction.

b. Convert $\sqrt{3}$ into an ultimately periodic continued fraction.