Dr. Z.'s Number Theory Homework assignment 3

1. Write down F_n for $1 \leq n \leq 10$.

2. List all the 1-2 walks from 0 to 7. Count them and make sure that you get $f_7 = F_8$.

3. Prove the following identities (using the fact that F_n is a linear combination of α^n and β^n where $\alpha, \beta = \frac{1 \pm \sqrt{5}}{2}$.

a.

$$F_{n+3} - F_n = 2F_{n+1} \quad .$$

b.

$$F_{n-2} + F_{n+2} = 3F_n \quad .$$

c.

$$F_{n+4} - F_{n+3} + 2F_{n+2} - 3F_{n+1} - 3F_n = 0 \quad .$$

4. Use the combinatorial model (in terms of paths) to prove that for every positive integer n,

$$F_{m+n+1} = F_{m+1}F_{n+1} + F_m F_n \quad .$$

5. Give a Zeilberger-style proof of the following identities, using the provided N_0

a.

$$F_{n+1}F_n - F_{n-1}F_{n-2} = F_{2n-1} \quad , \quad N_0 = 5 \quad .$$

b.

$$\sum_{i=1}^{2n-1} F_i F_{i+1} = F_{2n}^2 \quad , \quad N_0 = 5 \quad .$$