1. Write down $F_n$ for $1 \leq n \leq 10$.

2. List all the 1-2 walks from 0 to 7. Count them and make sure that you get $f_7 = F_8$.

3. Prove the following identities (using the fact that $F_n$ is a linear combination of $\alpha^n$ and $\beta^n$ where $\alpha, \beta = \frac{1 \pm \sqrt{5}}{2}$).
   a. $F_{n+3} - F_n = 2F_{n+1}$.
   b. $F_{n-2} + F_{n+2} = 3F_n$.
   c. $F_{n+4} - F_{n+3} + 2F_{n+2} - 3F_{n+1} - 3F_n = 0$.

4. Use the combinatorial model (in terms of paths) to prove that for every positive integer $n$,
   
   $$F_{m+n+1} = F_{m+1}F_{n+1} + F_mF_n$$.

5. Give a Zeilberger-style proof of the following identities, using the provided $N_0$
   a. $F_{n+1}F_n - F_{n-1}F_{n-2} = F_{2n-1}$, $N_0 = 5$.
   b. $\sum_{i=1}^{2n-1} F_iF_{i+1} = F_{2n}^2$, $N_0 = 5$. 