

Dr. Z.'s Number Theory Homework assignment 24

1. Evaluate the general continued fractions

a.

$$3 + \frac{2}{6 + \frac{2}{3}} \quad .$$

b.

$$2 + \frac{3}{1 + \frac{5}{2 + \frac{4}{5}}} \quad .$$

2. Convert the following rational numbers into simple continued fractions.

a. $\frac{6}{17}$ b. $\frac{50}{19}$ c. $\frac{100}{13}$

3. Express as a quadratic irrationality the following infinite continued fraction.

a.

$$x = [1, 4, 1, 4, 1, 4, 1, 4, \dots] \quad ,$$

where 1, 4 repeat for ever.

b.

$$x = [2, 3, 4, 2, 3, 4, 2, 3, 4, \dots] \quad ,$$

where 2, 3, 4 repeat for ever.

4. Find a representation in the form $a + b\sqrt{Q}$ for rational numbers a and b and positive integer Q , for the following infinite, ultimately periodic, continued fractions x .

(Hint: you should use what you got in problem 3.)

a.

$$x = [5, 1, 4, 1, 4, 1, 4, 1, 4, \dots] \quad ,$$

where 1, 4 repeat for ever.

b.

$$x = [5, 1, 2, 3, 4, 2, 3, 4, 2, 3, 4, \dots] \quad ,$$

where 2, 3, 4 repeat for ever.

5. a. Convert $\sqrt{5}$ into an ultimately periodic continued fraction.

b. Convert $\sqrt{3}$ into an ultimately periodic continued fraction.