1. Write down the generating function for the sequence, let’s call it \(a(n)\), for the number of partitions of \(n\) where each part shows up at most two times and whose largest part is \(\leq 4\). Use it to find \(a(i)\) for all \(1 \leq i \leq 6\).

2. Write down the generating function for the sequence, let’s call it \(a(n)\), for the number of partitions of \(n\) that are congruent to 2 or 3 modulo 5. Use it to find \(a(i)\) for \(1 \leq i \leq 7\).

3. Write down the generating function for the sequence, let’s call it \(a(n)\), for the number of partitions of \(n\) that are congruent to 1 or 5 modulo 6. Use it to find \(a(i)\) for \(1 \leq i \leq 8\).

4. Understand and be able to replicate, under test condition Euler’s proof that for every positive integer \(n\), the number of partitions of \(n\) into distinct parts equals the number of partitions of \(n\) into odd parts.

5

(a) Write down the sets \(\text{Odd}(8)\) and \(\text{Dis}(n)\).

(b) For each member of \(\text{Odd}(8)\) apply Glashier’s bijection in the odd \(\rightarrow\) distinct direction, and convince yourself that you got all the members of \(\text{Dis}(8)\).

(c) For each member of \(\text{Dis}(8)\) apply Glashier’s bijection in the distinct \(\rightarrow\) odd direction, and convince yourself that you got all the members of \(\text{Odd}(8)\).

6.

i. Apply Glashier’s bijection (in the odd \(\rightarrow\) distinct direction) to the odd partition \((5, 5, 5, 5, 3, 1, 1, 1)\) to get a distinct partition, call it \(\lambda\)

ii. Apply Glashier’s bijection (in the distinct \(\rightarrow\) odd direction) to the partition \(\lambda\) and show that you get \((5, 5, 5, 5, 3, 3, 1, 1, 1)\) back, as you should.

7.

i. Apply Glashier’s bijection (in the distinct \(\rightarrow\) odd direction) to the distinct partition \((5, 4, 3, 2, 1)\) to get a partition, call it \(\mu\)

ii. Apply Glashier’s bijection (in the odd \(\rightarrow\) distinct direction) to the partition \(\mu\) and show that you get \((5, 4, 3, 2, 1)\) back, as you should.