**Dr. Z.'s Number Theory Homework assignment 22**

**1.** Write down the generating function for the sequence, let's call it $a(n)$, for the number of partitions of $n$ where each part shows up at most two times and whose largest part is $\leq 4$. Use it to find $a(i)$ for all $1 \leq i \leq 6$.

**2.** Write down the generating function for the sequence, let's call it $a(n)$, for the number of partitions of $n$ that are congruent to 2 or 3 modulo 5. Use it to find $a(i)$ for $1 \leq i \leq 7$.

**3.** Write down the generating function for the sequence, let's call it $a(n)$, for the number of partitions of $n$ that are congruent to 1 or 5 modulo 6. Use it to find $a(i)$ for $1 \leq i \leq 8$.

**4.** Understand and be able to replicate, under test condition Euler's proof that for every positive integer $n$, the number of partitions of $n$ into distinct parts equals the number of partitions of $n$ into odd parts.

**5**

(a) Write down the sets $Odd(8)$ and $Dis(n)$.

(b) For each member of $Odd(8)$ apply Glashier's bijection in the odd $\rightarrow$ distinct direction, and convince yourself that you got all the members of $Dis(8)$.

(c) For each member of $Dis(8)$ apply Glashier's bijection in the distinct $\rightarrow$ odd direction, and convince yourself that you got all the members of $Odd(8)$.

**6.**

**i.** Apply Glashier's bijection (in the odd $\rightarrow$ distinct direction) to the odd partition $(5,5,5,5,5,3,3,1,1,1)$ to get a distinct partition, call it $\lambda$

**ii.** Apply Glashier's bijection (in the distinct $\rightarrow$ odd direction) to the partition $\lambda$ and show that you get $(5,5,5,5,5,3,3,1,1,1)$ back, as you should.

**7.**

**i.** Apply Glashier's bijection (in the distinct $\rightarrow$ odd direction) to the distinct partition $(5,4,3,2,1)$ to get an partition, call it $\mu$

**ii.** Apply Glashier's bijection (in the odd $\rightarrow$ distinct direction) to the partition $\mu$ and show that you get $(5,4,3,2,1)$ back, as you should.