

Dr. Z.'s Number Theory Homework assignment 19

1. Compute $\mu(n)$ for $1 \leq n \leq 20$.
2. Check empirically that $\sum_{d|105} \mu(d) = 0$.
3. State the Möbius inversion formula, and check it empirically for all $n \in Div(42)$
4. (Without peeking) prove that for all n , $\sum_{d|n} \mu(d) = 0$.
5. Express the Dirichlet series for $\mu(n)$ in terms of the Riemann Zeta function and prove it.
6. (Challenge) Prove that for every $\epsilon > 0$, you can find a constant C_ϵ such that

$$\sum_{i=1}^n \mu(i) \leq C_\epsilon n^{\frac{1}{2}+\epsilon} \quad .$$