Dr. Z.'s Number Theory Homework assignment 18

Version of Nov. 10, 2024 (thanks to Shreya Gosh)

1. Which of the following are perfect numbers? Explain!

a. 496 ; **b.** 100 ; **c.** 1000 ; **d.** 8128.

2. Using the Lucas-Lehmer test (no credit for other methods), show that $M_{11} = 2^{11} - 1 = 2047$ is **not** a Mersenne prime. **Note**: You may use a calculator (or computer) to compute S_9.

3. (Without peeking at your notes), prove that if p is a prime, and $2^p - 1$ is also a prime, then

$$2^{p-1} \cdot (2^p - 1)$$

is a perfect number.

4. (Without peeking at your notes), prove that if p and q are distinct odd primes,then pq can **not** be a perfect number.

5. (Without peeking at your notes), state precisely the Lucas-Lehmer test for testing whether M_p is a Mersenne prime.

6. (Without peeking at your notes), prove that if n is an integer that is not a prime, then $2^n - 1$ is not a prime either.