

## Solutions to Math 481 Attendance QUIZ for Lecture 7

1. If you roll  $n$  fair standard dice, and look at the sum. What is the expectation? What is the variance?

**Sol. of 1:** Let's first find the expectation and variance of rolling **one** fair standard die.

$$E[X] = \frac{1}{6} \cdot \left( \sum_{i=1}^6 i \right) = \frac{1}{6} \cdot \frac{6 \cdot 7}{2} = \frac{7}{2}$$

$$\begin{aligned} \text{Var}(X) &= \frac{1}{6} \left( \sum_{i=1}^6 \left( i - \frac{7}{2} \right)^2 \right) = \\ &= \frac{1}{6} \left( \left( \frac{5}{2} \right)^2 + \left( \frac{3}{2} \right)^2 + \left( \frac{1}{2} \right)^2 + \left( -\frac{1}{2} \right)^2 + \left( -\frac{3}{2} \right)^2 + \left( -\frac{5}{2} \right)^2 \right) = \frac{35}{12} . \end{aligned}$$

$$E[X_1 + \dots + X_n] = n \cdot E[X] = \frac{7n}{2} .$$

(Note that here we didn't have to use the fact that they were independent)

Since the  $n$  rolls are **independent**, and calling the  $i$ -th roll  $X_i$  we have

$$\text{Var}(X_1 + \dots + X_n) = n \text{Var}(X) = \frac{35n}{12} .$$

**Ans. to 1:** The expectation is  $\frac{7n}{2}$  and the variance is  $\frac{35n}{12}$  .

2. If you toss three coins with  $\Pr(\text{Heads}) = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ , what is the expected number of Heads? What is the variance?

**Sol. of 2:** Let  $X_1, X_2, X_3$  be the respective random variable.

Recall that for a single coin toss with a coin with  $\Pr(\text{Heads}) = p$ ,

$$E[X] = p \cdot 1 + (1 - p) \cdot 0 = p$$

$$\text{Var}(X) = p \cdot (1 - p)^2 + (1 - p) \cdot (0 - p)^2 = p(1 - p) .$$

The total number of Heads is  $X = X_1 + X_2 + X_3$ .

We have, by *linearity of expectation*

$$E[X] = E[X_1] + E[X_2] + E[X_3] = \frac{1}{4} + \frac{1}{2} + \frac{3}{4} = \frac{3}{2} .$$

We have, by *linearity of variance* (for independent events)

$$\text{Var}(X) = \text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) = \frac{1}{4} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{4} = \frac{5}{8} \quad .$$

**Ans. to 2:** The expectation of the total number of Heads is  $\frac{3}{2}$ , and its variance is  $\frac{5}{8}$ .