

## Solutions to Math 481 Attendance QUIZ for Lecture 4

1. Suppose that  $X$  is a random variable taking the values  $-2, -1, 0, 1, 2$  each with equal probability, and that  $Y = X^3$ . Find the joint distribution and the marginal distributions of  $X$  and  $Y$ , and also the conditional distribution of  $X$  given (a)  $Y = 0$  (b)  $Y = 1$

**Sol. of 1**

$$P(X = -2) = \frac{1}{5} \quad , \quad P(X = -1) = \frac{1}{5} \quad , \quad P(X = 0) = \frac{1}{5} \quad , \quad P(X = 1) = \frac{1}{5} \quad , \quad P(X = 2) = \frac{1}{5} \quad .$$

$$P(Y = -8) = \frac{1}{5} \quad , \quad P(Y = -1) = \frac{1}{5} \quad , \quad P(Y = 0) = \frac{1}{5} \quad , \quad P(Y = 1) = \frac{1}{5} \quad , \quad P(Y = 8) = \frac{1}{5} \quad .$$

Of course once  $X$  is known,  $Y$  can only be one thing. So the **joint frequency distribution** of  $X$  and  $Y$  is the square-matrix

|          | $Y = -8$ | $Y = -1$ | $Y = 0$ | $Y = 1$ | $Y = 8$ |
|----------|----------|----------|---------|---------|---------|
| $X = -2$ | 0.2      | 0        | 0       | 0       | 0       |
| $X = -1$ | 0        | 0.2      | 0       | 0       | 0       |
| $X = 0$  | 0        | 0        | 0.2     | 0       | 0       |
| $X = 1$  | 0        | 0        | 0       | 0.2     | 0       |
| $X = 2$  | 0        | 0        | 0       | 0       | 0.2     |

Since  $Y$  depends on  $X$  (and vice-versa), the *marginal distributions* of  $X$  is the same as the distribution of  $X$ . Also the marginal distribution of  $Y$  is the same as the distribution of  $Y$ .

Finally, the *conditional distribution*  $P(X = x|Y = 0)$  is as follows:

$$P(X = -2|Y = 0) = 0 \quad , \quad P(X = -1|Y = 0) = 0 \quad , \quad P(X = 0|Y = 0) = 1 \quad ,$$

$$P(X = 1|Y = 0) = 0 \quad , \quad P(X = 2|Y = 0) = 0 \quad .$$

and the *conditional distribution*  $P(X = x|Y = 1)$  is as follows:

$$P(X = -2|Y = 1) = 0 \quad , \quad P(X = -1|Y = 1) = 0 \quad , \quad P(X = 0|Y = 1) = 0 \quad ,$$

$$P(X = 1|Y = 1) = 1 \quad , \quad P(X = 2|Y = 1) = 0 \quad .$$

**Remark:** This problem was only meant to make you familiar with the concepts. The computations are trivial. If  $Y = X^2$  like in ex. 3.4 of today's homeork, it is a little more interesting, since while  $X$  and  $Y$  are **dependent** they are not *one – to – one*, so the matrix describing the *joint frequency distribution* is not a diagonal matrix, like in this **very boring** problem.