

NAME: (print!)

E-Mail address:

MATH 477 (3), Dr. Z. , Final Exam, Monday, Dec. 18, 2017, 8:00-11:00am, PH 111

PUT The FINAL ANSWER TO EACH PROBLEM IN THE AVAILABLE BOX

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tot:

EXPLAIN EVERYTHING! Only simple calculators are allowed. You need the Z table.

1. (12 pts.) Suppose that it is known that the amount of gold dug in a Gold mine during one day is a random variable with mean 100 kg and variance 25 kg^2 . what can be said about the probability that a day's production will be between 80 and 120 kg?

ans.

2. (12 pts.) The moment generating function of a certain random variable, X , is $M_X(t) = e^{t^2/2+t^3/6}$. Calculate the third moment.

ans.

3. (12 pts.) In the sample space $\{-1, 0, 1, 2\}$, with $Pr(-1) = Pr(0) = Pr(1) = Pr(2) = \frac{1}{4}$ define the random variables $X(i) = i, Y(i) = -i^2$. Find the correlation $\rho(X, Y)$.

ans.

4. (12 pts.) The return on three investments, X and Y, Z , follows the joint probability density function

$$f(x, y, z) = \begin{cases} 8xyz & , \text{ if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \text{ and } 0 \leq z \leq 1; \\ 0 & , \text{ otherwise.} \end{cases}$$

Calculate $E[6X^2Y^2Z]$.

ans.

5. (12 pts.) A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy.

Let X denote the proportion of employees who purchase the basic policy, and let Y the proportion of the employees who purchase the supplemental policy. Let X and Y have joint density function $f(x, y) = 2(x + y)$ on the region where the density is positive.

Given that 30% of the employees buy the basic policy, determine the probability that fewer than 20% buy the supplemental policy.

ans.

6. (8 pts.) State the axioms of probability for finite sample spaces.

7. (12 pts.) Let X be a random variable with probability density function

$$f(x) = \begin{cases} 3x^2 & , \text{if } 0 \leq x \leq 1; \\ 0 & , \text{otherwise.} \end{cases}$$

Find the standard-deviation of X .

ans.

8. (12 pts.) The monthly profit of Company I can be modeled by a continuous random variable with density function $f(x)$. Company II has a monthly profit that is the cube of that of Company I.

Determine the probability density function, call it $g(y)$, of the monthly profit of Company II.

ans.

9. (12 pts.) The probability that a randomly chosen dog is a biter is 0.2. Dogs who are biters are three times as likely to be barkers as those who do not bite.

What is the conditional probability that a randomly chosen dog is a biter, given that it is a barker?

ans.

10. (12 pts.) What is the probability that among 3 people, at least two of them have the same birth-month? (Assuming (a little stupidly, since Aug. is more likely than Feb.) that all 12 months are equally likely).

ans.

11. (12 pts.) It is known that the probability of being lazy equals twice the probability of being smart, and the probability of being smart is twice the probability of being strong. All three traits are independent of each other. If the probability of being

lazy **and** smart **and** strong is $\frac{8}{1000}$, what is the probability of being lazy?

ans.

12. (12 pts.) Suppose that $Var(X) = 1, Var(Y) = 3, Var(Z) = 5, Var(X + Y) = 6, Var(X + Z) = 10, Var(Y + Z) = 10$. Find $Var(X - Y + Z)$.

ans.

13. (12 pts.) Let X_1, \dots, X_{300} be independent random variables, each with (the same) density function given by:

$$f(x) = \begin{cases} 2x & , \text{ if } 0 \leq x \leq 1; \\ 0 & , \text{ otherwise} \end{cases} ,$$

and let $X = \sum_{i=1}^{300} X_i$.

Use the Central Limit Theorem to estimate the probability that $X < 202$. You can leave your answer in terms of the Φ function (the cdf of the standard normal distribution)

ans.

14. (12 pts.) (a) (6 points) State Chebychev's Inequality ; (b) (6 points) State the weak Law of Large Numbers.

15. (12 pts.) You toss a fair coin three times.

If you get 3 Heads you win 2000 dollars. If you get 2 Heads you win 1000 dollars. If you get 1 Heads you lose 600 dollars. If you get 0 Heads you lose 1400 dollars.

What is your expected gain, if it is known that you did **not** get three Heads.

ans.

16. (12 points) State (3 points) and prove (9 points) Markov's inequality.

17. (12 pts.) You are visiting the Royal Gardens in London, England, that is a huge maze. There are no people to ask directions, and no maps (or gps).

The garden has two statues, Queen Victoria and Queen Mary. You are currently next to Queen Victoria, and you are desperate to get out of the maze.

- If you are at the Queen Victoria statue, there are two paths. One path leads out of the maze, and takes 9 minutes to walk, the second takes 3 minutes to walk and takes you to the statute of Queen Mary. You are equally likely to take either path (if you are currently at Queen Victoria).

- If you are at the Queen Mary statue, there are two paths. One path leads out of the maze, and takes 12 minutes, the second takes 6 minutes and takes you to the statute of Queen Victoria. You are equally likely to take either path (if you are currently at Queen Mary).

What is the expected time of getting out of the maze, if you are currently next to the statute of Queen Victoria?

ans.
