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MATH 477 (3), Dr. Z. , Exam 1, Thurs., Oct. 19, 2017, 8:40-10:00am, HLL 116

PUT The FINAL ANSWER TO EACH PROBLEM IN THE AVAILABLE BOX

Do not write below this line

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1. 10 (out of 10)
2. 10 (out of 10)
3. 10 (out of 10)
4. 10 (out of 10)
5. 10 (out of 10)
6. 10 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 10 (out of 10)
10. 10 (out of 10)

EXPLAIN EVERYTHING! Only simple calculaculators are allowed.

100

Exc.

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1. (10 pts.) The number of injury claims per month is modeled by a random variable N with

$$P\{N = n\} = \frac{6}{\pi^2 n^2}, \quad where \quad n \geq 1 \quad ,$$

(and 0 otherwise). Determine the probability of exactly one claim during a particular month, given that there have been at most three claims during that month.

ans. 36/49 ✓ 10

$$P(X=1 \mid X \leq 3) = \frac{P(X=1)}{P(X \leq 3)} = \frac{\cancel{\frac{6}{\pi^2}}\ (1)}{\cancel{\frac{6}{\pi^2}}\ \left(\frac{49}{36}\right)} = \frac{36}{49}$$

$$P(X=1) = \frac{6}{\pi^2}$$

$$P(X \leq 3) = \frac{6}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}\right) = \frac{6}{\pi^2}\ \frac{49}{36}$$

2. (10 points) The alphabet set of certain language is $\{A, B, C\}$. The frequency of A is 0.5, the frequency of B is 0.3, the frequency of C is 0.2. If you make a random word of 9 letters (and draw each letter independently of the other ones), what is the probability that is has 3 A's, 3 B's, and 3 C's ?

ans. 0.04536

10

Multinomial coefficient

$$\frac{(3+3+3)!}{3!\,3!\,3!} \cdot (0.5)^3 (0.3)^3 (0.2)^3 =$$

$$1680\ (0.125)(0.027)(0.008)$$

$$= 0.04536$$

3. (10 points) A company agrees to accept the highest of five sealed bids on property. The five bids are regarded as five independent random variables with common **probability density function**

$$f(x) = \begin{cases} 6x^5 & , 0 < x < 1 \quad ; \\ 0 \; , & otherwise. \end{cases}$$

What is the expected value of the accepted bid?

ans. $\boxed{30/31}$

10

$$P(\tilde{X} < x) = P(X_1 < x)P(X_2 < x)\ldots P(X_5 < x)$$
$$= P(X < x)^5$$

where $\tilde{X}$ is the value of the max bid.

$$P(X < x) = \int_0^x 6x'^5 dx' = x^6$$

$$P(\tilde{X} < x)' = f(\tilde{x}) = 5P(X<x)^4 \cdot P(X<x)'$$
$$f(\tilde{x}) = 5(x^6)^4 \cdot f(x)$$
$$f(\tilde{x}) = 5x^{24} \cdot 6x^5$$
$$f(\tilde{x}) = 30x^{29}$$

$$E[\tilde{x}] = \int_0^1 x \cdot 30x^{24} dx = \frac{30}{31} x^{30}\Big|_0^1 = \frac{30}{31}$$

4. (10 points) In a certain class, there are Math Majors, CS majors, double-Math-CS majors, and students who major neither in math nor in CS.
• the probability that someone is only a math major (and not a CS major) is 0.4
• the probability that someone is only a CS major (and not a Math major) is 0.2
• the probability that someone is a double-math-CS major is 0.1
Find the conditional probability that someone is math major if he or she is not a CS major.

ans. $4/7$ ✓ 10

$P(MC^c) = 0.4$

$P(M^cC) = 0.2$

$P(MC) = 0.1$

Since $1 = P(MC^c) + P(M^cC) + P(MC) + P(M^cC^c)$

$1 = 0.4 + 0.2 + 0.1 + P(M^cC^c)$

$P(M^cC^c) = 0.3$

$P(C^c) = P(MC^c) + P(M^cC^c) = 0.7$

$$P(M|C^c) = \frac{P(MC^c)}{P(C^c)} = \frac{0.4}{0.7} = \frac{4}{7}$$

5. (10 points) In a certain country $\frac{1}{5}$ of the population are college graduates. $\frac{9}{10}$ of the college graduates wear glasses, but only $\frac{1}{10}$ of the non-college graduates wear glasses. You see a random person in the street wearing glasses, what is the probability that she or he is a college graduate?

ans. 9/13

10

Find $P(C|G)$

$P(C) = \frac{1}{5} \rightarrow P(C^c) = \frac{4}{5}$

$P(G|C) = \frac{9}{10}$

$P(G|C^c) = \frac{1}{10}$

$P(G) = P(C)P(G|C) + P(C^c)P(G|C^c)$

$P(G) = \frac{1}{5} \cdot \frac{9}{10} + \frac{4}{5}\frac{1}{10} = \frac{9}{50} + \frac{4}{50} = \frac{13}{50}$

$$P(C|G) = \frac{P(G|C)P(C)}{P(G)} = \frac{\frac{1}{5}\cdot\frac{9}{10}}{\frac{13}{50}} = \frac{\frac{9}{50}}{\frac{13}{50}} = \frac{9}{13}$$

6. (10 points) State the three axioms of probability (for finite sample spaces).

Let Ω denote the sample space set, and A and B denote subsets of Ω

(I) $P(\Omega) = 1$

(II) $0 \leq P(A) \leq 1$

(III) $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \emptyset$

10

7. (10 points) The number of letters that I find in my mailbox, in any given day, has a Poisson distribution with mean 4. Today my wife told me that I got strictly more than two letters. What is the probability that I got exactly three letters?

ans. 0.25642165 97, .. ✓ 9.9999

$\lambda = 4$

find
$P(X=3 \mid X>2)$

$$P(X=3) = e^{-4}\frac{4^3}{3!} = \frac{32}{3}e^{-4} = 0.196$$

$$P(X>2) = \sum_{i=3}^{\infty} e^{-4}\frac{4^i}{i!} = 1 - e^{-4}\sum_{i=0}^{2}\frac{4^i}{i!} = 1 - e^{4}(1+4+8)$$
$$= 0.762$$

$$P(X=3 \mid X>2) = \frac{\frac{32}{3}e^{-4}}{1-e^{-4}\sum_{i=0}^{3}\frac{4^i}{i!}} = \frac{0.195}{0.762} = 0.256$$

8. (10 points) A study is being conducted in which the health of four independent groups, each consisting of five policyholders, is monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independent of the other participants). What is the probability that there are at least two groups where at least four participants completed the study?

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ans. 0.941 - ...

0.941758 ...

9.9

$P(\text{Stay}) = 0.8$

$$P(X \geq 4) = \sum_{i=4}^{5} \binom{5}{i} (0.8)^i (0.2)^{5-i}$$

$$= 0.327 + 0.4096 = 0.7366$$

$P(\text{Complete Group}) = 0.7366$

$$P(Y \geq 2) = \sum_{i=2}^{4} \binom{4}{i} (0.7366)^i (.2634)^{4-i}$$

$$= (0.226) + (0.421) + (0.294)$$

$$= 0.941$$

9. (10 points) The probability that you win i dollars $(1 \leq i \leq 3)$ is proportional to $\frac{1}{i^2}$ (and otherwise it is 0). Let X be the amount won, find the standard deviation of X.

ans. 0.624 ✓ 10

$$1 = \frac{c}{1^2} + \frac{c}{2^2} + \frac{c}{3^2} = \frac{49}{36}c$$

$$c = \frac{36}{49}$$

$$E[X] = \frac{36}{49}\left[\frac{1}{1^2}\cdot 1 + \frac{1}{2^2}\cdot 2 + \frac{1}{3^2}\cdot 3\right] = 1.347$$

$$E[X^2] = \frac{36}{49}\left[1\cdot 1 + \frac{1}{2^2}2^2 + \frac{1}{3^2}3^2\right) = 2.204$$

$$Var[X] = 2.204 - 1.347^2 = 0.389$$

$$SD(X) = \sqrt{0.389} = 0.624$$

10. (10 points) You toss a coin, whose probability of Tails is 0.3, 1000 times. Let X be the random variable "Number of Heads in the first 700 tosses Plus three times the Number of Tails in the last 300 tosses". Find $E[X]$.

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ans. 760

10

$P(\text{Heads}) = 0.7$

$P(\text{Tails}) = 0.3$

$E[X_1] = np(\text{Heads}) = 700(0.7) = 490$

$E[X_2] = 3np(\text{Tails}) = 3 \cdot 300(0.3) = 270$

$E[X] = E[X_1] + E[X_2] = 760.$