Solutions to Math 477 REAL QUIZ #9

1. (5 points) You go to a casino in St. Petersburg where you have a chance of 10^{-6} of winning one hundred million rubles, and a chance of $1 - 10^{-6}$ of losing 10 rubles. You do it for n days, and each time is independent of the other times. If X is the random variable denoting your gain, what is the probability generating function? What is $E[X]$? What is $Var(X)$?

Sol. to 1: The prob. generating function for *one* day is

$$10^{-6}x^{10^8} + (1-10^{-6})x^{-10} \quad ,$$

Hence for n days it is

$$(10^{-6}x^{10^8} + (1-10^{-6})x^{-10})^n \quad .$$

The expected gain in **one** day is

$$10^{-6} \cdot 10^8 + (1-10^{-6}) \cdot (-10) = 90.00001 \quad .$$

Hence the expected gain in n days is $90.00001\,n$.

The variance of the gain in **one** day is

$$10^{-6} \cdot (10^8 - 90.00001)^2 + (1-10^{-6}) \cdot (-10 - 90.00001)^2 \quad APPROXIMATELY \quad 10^{10} \quad ,$$

Hence the variance of the total gain in n days is about $10^{10}n$.

2. (5 points) Let X and Y be the number of hours that a randomly selected person watches movies and sports events, respectively, during a three-month period. The following information is known about X and Y.

$$E[X] = 30 \quad , \quad E[Y] = 30 \quad , \quad Var(X) = 20 \quad , \quad Var(Y) = 20 \quad , \quad Cov(X,Y) = 30 \quad .$$

Four hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these four hundred people watch movies or sports events this three month period.

Approximate the value of $P(T < 26000)$.

Sol. to 2: We first need $Var(X+Y)$.

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y) = 20 + 20 + 2 \cdot 30 = 100 \quad ,$$

hence $\sigma^{=}100$ and $\sigma = 10$. Also

$$E[X+Y] = E[X] + E[Y] = 30 + 30 = 60 \quad .$$

Hence $\mu = 60$ and $\sigma = 10$. By the **Central limit theorem**

$$T = \sum_{i=1}^{400} T_i \quad ,$$

is **approximately** a normal random variable with mean $n\mu = 400 \cdot 60 = 24000$ and standard deviation $\sqrt{n}\sigma = \sqrt{400} \cdot 10 = 200$. Hence

$$P\{T < 260000\} = P\{T - 24000 < 26000 - 24000\} = P\{\frac{T - 24000}{200} < \frac{26000 - 24000}{200}\} = P\{\frac{T - 24000}{200} < 10\} \quad ,$$

is approximately $P\{Z < 10\}$ that equals $\Phi(10)$ that is practically 1.