Solutions to Math 477 REAL QUIZ #6

1. (5 points) The number of years a radio functions is exponentially distributed with mean 10 years. If Jones buys a used radio, what is the probability that it will be working an additional 20 years?

Sol. to 1: The **parameter** is $\lambda = \frac{1}{10}$. For this problem it is better to use $\bar{F}(x) = P\{X \geq x\}$, the **complementary cdf**, whose formula is extremely simple

$$\bar{F}(x) = e^{-\lambda x} \quad .$$

Hence, the desired probability is

$$P\{X \geq 20\} \; = \; e^{-\frac{1}{10}\cdot 20} = e^{-2} = 0.13533528\ldots \quad .$$

Ans. to 1: The probability that Jones' radio will be working an additional 20 years is $e^{-2} = 0.13533528\ldots$.

2. (5 points) The working lifetime, in years, of a particular model of bread maker is normally distributed with mean 2 and standard deviation $\sqrt{2}$.

Calculate the 40-th percentile of the working lifetime, in years.

Sol. to 2:Let a be the 40-th percentile of the working life of the bread maker. We know that

$$P\{X \leq a\} = 0.4 \quad .$$

Rectall that

$$Z = \frac{X - \mu}{\sigma} \quad ,$$

is a **standard normal r.v.**, $N(0,1)$, for which the Z table is applicable.

In this problem $\mu = 2$ and $\sigma = \sqrt{2}$. Subtracting $\mu = 2$ from both sides of $X \leq a$, we get

$$P\{X - 2 \leq a - 2\} = 0.4 \quad .$$

Dividing by $\sigma = \sqrt{2}$ both sides of the inequailtiy $X - 2 \leq a - 2$, we get

$$P\{\frac{X-2}{\sqrt{2}} \leq \frac{a-2}{\sqrt{2}}\} = 0.4 \quad .$$

Hence

$$\Phi(\frac{a-2}{\sqrt{2}}) \; = \; 0.4.$$

From the Z-table, we get

$$\Phi(-0.255) = 0.4 \quad .$$

Hence

$$\frac{a-2}{\sqrt{2}} \; = \; -0.255 \quad ,$$

and hence

$$a = 2 - 0.255 \cdot \sqrt{2} = 1.6393 \ldots \quad .$$

Ans. to 2: The probability that a random bread-maker would break down before the young age of $1.639375542\ldots$ years is %40.

Comments:

1. You should always carry with you a copy of the table of Φ. Here it is

`http://sites.math.rutgers.edu/~zeilberg/math477/Ztable.pdf` .

But if for some reason you don't have it, you should write the answer as

$2 + \Phi^{-1}(0.4) \cdot \sqrt{2}$, where $\Phi(z)$ is the cdf of $N(0,1)$ that is famously given by the formula

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2} \, dx \quad .$$

2. The table is not that accurate. A better table gives you

$$\Phi^{-1}(0.4) \; = \; -0.2533475 \quad ,$$

giving a more accurate answer of 1.641712530 .