

Solutions to Math 477 REAL QUIZ #4

1. (5 points) An actuary discovers that policyholders are four times as likely to file three claims as to file four claims. If the number of claims filed has a Poisson distribution, what is the second moment of the number of claims filed?

Sol. to 1: We are told that

$$\frac{P\{X = 3\}}{P\{X = 4\}} = 4 \quad .$$

Let λ be the parameter of the Poisson random variable. Since the probability mass function of a Poisson random variable is

$$P\{X = k\} = e^{-\lambda} \frac{\lambda^k}{k!} \quad ,$$

we have

$$\frac{e^{-\lambda} \frac{\lambda^3}{3!}}{e^{-\lambda} \frac{\lambda^4}{4!}} = 4 \quad .$$

Simplifying (the $e^{-\lambda}$ cancels out).

$$\frac{\frac{\lambda^3}{3!}}{\frac{\lambda^4}{4!}} = \frac{\frac{\lambda^3}{6}}{\frac{\lambda^4}{24}} = \frac{4}{\lambda} \quad .$$

Hence

$$\frac{4}{\lambda} = 4 \quad .$$

Giving $\lambda = 1$. The second moment of a Poisson random variable is $\lambda(\lambda + 1)$. Hence the second moment is $1 \cdot (1 + 1) = 2$.

Ans. to 1: The second moment of the number of claims filed is 2.

2. (5 points) A certain loaded die has $P(X = 1) = 0.5, P(X = 2) = 0.1, P(X = 3) = 0.1, P(X = 4) = 0.1, P(X = 5) = 0.1, P(X = 6) = 0.1$. If you roll it 100 times, what is the expected number of times it lands on even integer? What is the standard-deviation?

Sol. to 2: The probability that it lands on an even integer in **one throw** is

$$P\{X = 2\} + P\{X = 4\} + P\{X = 6\} = 0.1 + 0.1 + 0.1 = 0.3 \quad .$$

This is an example of a **binomial random variable** $B(n, p)$ with $n = 100$ and $p = 0.3$. Hence the **expectation** is $np = 100 \cdot 0.3 = 30$. The **variance** is $np(1 - p) = 100 \cdot 0.3 \cdot 0.7 = 21$, hence the **standard deviation** is $\sigma = \sqrt{21} = 4.582575695 \dots$.

Ans. to 2: The expected number of times it lands on an even integer is 30, the standard deviation is $\sqrt{21} = 4.582575695 \dots$.