Solutions to Math 477 REAL QUIZ #3

1. (3 points) Let X be the winnings of a gambler and assume that

$$P\{X=1\}=0.1 \quad ; \quad P\{X=2\}=0.4 \quad ;$$

$$P\{X=-1\}=0.3 \quad ; \quad P(X=-2)=0.2 \quad ;$$

(i) Compute the conditional probability that gambler wins i, for $i=1,2$, given that he wins a positive amount. (ii) Find $E[X]$, his expected winning.

Sol. to 1(i): For future reference, let's compute first $P\{X>0\}$

$$P\{X>0\}=P\{X=1\}+P\{X=2\} \ = \ 0.1+0.4=0.5 \quad .$$

Now we have

$$P\{X=1|X>0\}=\frac{P\{X=1\}}{P\{X>0\}}=\frac{0.1}{0.5}=0.2 \quad ,$$

$$P\{X=2|X>0\}=\frac{P\{X=2\}}{P\{X>0\}}=\frac{0.4}{0.5}=0.8 \quad .$$

Ans. to 1(i): $0.2, 0.8$.

Sol. to 1(ii):

$$E[X]=P\{X=1\}\cdot 1+P\{X=2\}\cdot 2+P\{X=-1\}\cdot(-1)+P\{X=-2\}\cdot(-2)$$

$$(0.1)\cdot 1+(0.4)\cdot 2+(0.3)\cdot(-1)+(0.2)\cdot(-2)=0.1+0.8-0.3-0.4=0.2 \quad .$$

Ans. to 1(ii): $E[X]=0.2$.

2. (4 points) The number of injury claims per month is modeled by a random variable N with

$$P\{N=n\} \ = \ \frac{4}{(n+1)\,(n+2)\,(n+3)}, \quad where \quad n\geq 0 \quad .$$

Determine the probability of at least one claim during a particular month, given that there have been at most two claims during that month.

Sol. to 2:

$$P(N\leq 1|N\leq 2)=\frac{P\{1\leq N\leq 2\}}{P\{0\leq N\leq 2\}}=\frac{P\{N=1\}+P\{N=2\}}{P\{N=0\}+P\{N=1\}+P\{N=2\}} \quad .$$

Now

$$P\{N=0\}=\frac{4}{(0+1)\,(0+2)\,(0+3)}=\frac{2}{3} \quad ,$$

$$P\{N=1\} = \frac{4}{(1+1)\,(1+2)\,(1+3)} = \frac{1}{6} \quad ,$$

$$P\{N=2\} = \frac{4}{(2+1)\,(2+2)\,(2+3)} = \frac{1}{15} \quad .$$

Hence

$$P(N \leq 1 | N \leq 2) \;=\; \frac{1/6 + 1/15}{2/3 + 1/6 + 1/15} = \frac{7}{27} \quad .$$

Ans. to 2: The probability of at least one claim during a particular month, given that there have been at most two claims during that month is $\frac{7}{27}$.

3. (3 points) An n-faced fair die, marked with $1, 2, \ldots n$ is rolled. What are the Expected number of the cube of the number of dots of landed face?

Sol. to 3: We need

$$\frac{1}{n}\left(\sum_{i=1}^{n} i^3\right) \quad .$$

By a famous and beautiful formula

$$\sum_{i=1}^{n} i^3 \;=\; \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4} \quad .$$

Hence

Ans. to 3: $\frac{n(n+1)^2}{4}$.