**Solutions to Math 477 REAL QUIZ #2**

**1.** (5 points) In a certain class, the only bad habits are smoking and drinking.

• the probability that someone only drinks is 0.3

• the probability that someone only smokes is 0.1

• the probability that someone smokes and drinks is 0.05

Find

(i) The conditional probability that someone doesn't smoke if it is known that he doesn't drink;

(ii) The conditional probability that someone smokes if it is known that he doesn't drink;

(iii) The conditional probability that someone smokes if it is known that he drinks.

**Sol. to 1**: We have

$$P(DS^c) = 0.3 \quad , \quad P(D^cS) = 0.1 \quad , \quad P(DS) = 0.05 \quad ,$$

hence since everything must add-up to 1,

$$P(D^cS^c) \; = \; 1 - (0.3 + 0.1 + 0.05) \; = \; 0.55 \quad .$$

For future reference, we also need $P(D) \; = \; P(DS) + P(DS^c) \; = \; 0.05 + 0.3 \; = \; 0.35$ and $P(D^c) \; = \; 1 - P(D) = 1 - 0.35 = 0.65$.

Now we are ready to answer the questions.

**(i)**:

$$P(S^c|D^c) \; = \; \frac{P(D^cS^c)}{P(D^c)} \; = \; \frac{0.55}{0.65} \; = \; \frac{11}{13} \quad .$$

**(ii)**:

$$P(S|D^c) \; = \; \frac{P(D^cS)}{P(D^c)} \; = \; \frac{0.1}{0.65} \; = \; \frac{2}{13} \quad .$$

**Note**: another way of doing is to note that $P(S|D^c) \; = \; 1 - P(S^c|D^c)$ , and since we already know (from part (i)) that $P(S^c|D^c) \; = \; \frac{11}{13}$ we get $1 - \frac{11}{13} \; = \; \frac{2}{13}$.

**(iii)**:

$$P(S|D) \; = \; \frac{P(DS)}{P(D)} \; = \; \frac{0.05}{0.35} = \frac{1}{7} \quad .$$

**2.** ( 5 points) It is known that the probability of being smart is twice the probability of being handsome, and the probability of being handsome is four times the probability of being nice. All

three traits are independent of the other ones. If the probability being smart and handsome and nice is %3.2, what are the probabilities of being smart?, of being handsome?, of being nice?

**Sol. to 2**: Let $N$, $H$ and $S$ be the events "being nice", "being handsome", and "being smart", respectively. We are told that $P(H) = 4P(N)$ and $P(S) = 2P(H)$. Let $P(N) = x$, so

$$P(N) = x \quad , \quad P(H) = 4x \quad , \quad P(S) = 2 \cdot 4x = 8x \quad .$$

Since the three traits are **independent**, we have

$$P(NHS) = P(N) \cdot P(H) \cdot P(S) = x(4x)(8x) = 32x^3 \quad .$$

On the other hand, the problem tells us that $P(NHS) = 3.2/100 = 32/1000$. We have to solve

$$32x^3 = \frac{32}{1000} \quad .$$

Dividing both sides by 32, we get:

$$x^3 = \frac{1}{1000} \quad .$$

Hence

$$x = \frac{1}{10} \quad .$$

Going back above, we have

$$P(N) = \frac{1}{10} \quad , \quad P(H) = \frac{4}{10} = \frac{2}{5} \quad , \quad P(S) = \frac{8}{10} = \frac{4}{5} \quad .$$

**Ans. to 2**: The probability of being smart is $\frac{4}{5}$, of being handsome is $\frac{2}{5}$, of being nice is $\frac{1}{10}$.