Solutions to Dr. Z.'s Math 477 REAL QUIZ #1

1.(5 points) Out of a large group of students it is found that one half take both calculus and algebra, and one-quarter take neither of them. The probability that a student takes calculus is twice the probability that a student takes algebra.

Determine the probability that a randomly chosen member of this group takes algebra?

Sol. to 1: Let

- A be the event "taking algebra"

- C be the event "taking calculus"

Let $P(A) = x$. The problem tells us

$$P(A) = x \quad , \quad P(C) = 2x \quad , \quad P(AC) = \frac{1}{2} \quad , \quad P(A^c C^c) = \frac{1}{4} \quad .$$

By the famous **Principle of Inclusion-Exclusion** (for two events)

$$P(A^c C^c) = 1 - P(A) - P(C) + P(AC) \quad .$$

Plugging-in the data of the problem:

$$\frac{1}{4} = 1 - x - 2x + \frac{1}{2} \quad .$$

Simplifying:

$$3x = 1 + \frac{1}{2} - \frac{1}{4} = \frac{5}{4} \quad .$$

Hence $x = \frac{5}{12}$.

Ans. to 1: The probability that a randomly chosen member of this group takes algebra is $\frac{5}{12}$.

Comment added Sept. 21, 2017, 3:25pm Jason Ghaugan noticed that the problem is "impossible", and he won a dollar. It is 'virtual reality'. In real life $P(AC)$ is always $\leq$ than $P(A)$, since AC is a subset of A. But in this problem it happens to be larger, $P(AC) = \frac{1}{2}$ (from the data), while the answer was $P(A) = \frac{5}{12}$. It follows that in this problem $P(AC^c) = P(A) - P(AC) = \frac{5}{12} - \frac{1}{2} = -\frac{1}{6}$. OMG! a negative probability! Jason got a dollar, but anyone who got $P(A) = \frac{5}{12}$ got full credit, even though this problem can never come up in real life.

2.(5 points) If you have a box (where you can't see the inside) consisting of 10 red balls, 20 green balls, and 30 yellow balls, and you draw 10 balls (without replacing them). What is the probability that you would pick 3 red balls, 4 green balls, and 3 yellow balls?

Sol. to 2.: There are altogether $10 + 20 + 30 = 60$ balls in the box.

The **sample space** consists of all ten-element subsets so $|S| = \binom{60}{10}$.

The **event** is the **Cartesian prouduct** of three **independent** events:

- 3-element subsets of the set of 10 red balls, whose number of elements is $\binom{10}{3}$.

- 4-element subsets of the set of 20 green balls, whose number of elements is $\binom{20}{4}$.

- 3-element subsets of the set of 30 green balls, whose number of elements is $\binom{30}{3}$.

Hence $|E|$ is the **product** of the above three, namely $\binom{10}{3}\binom{20}{4}\binom{30}{3}$.

Hence the desired probability $|E|/|S|$ is

$$\frac{|E|}{|S|} = \frac{\binom{10}{3}\binom{20}{4}\binom{30}{3}}{\binom{60}{10}} \quad .$$

Ans. The probability that you would pick 3 red balls, 4 green balls, and 3 yellow balls is $\frac{\binom{10}{3}\binom{20}{4}\binom{30}{3}}{\binom{60}{10}}$.

Note: According to Maple, this equals $\frac{14000}{447161} = 0.03130863380\ldots$, but I don't expect you to evaluate it during a quiz or test.