Solutions to Math 477 "QUIZ" for Lecture 9

1. The number of days an employee is sick each year is modeled by a Poisson distribution with mean 10. The number of sick days in different years are mutually independent. Calculate the probability that an employee is sick more than 35 days in a three-year-period. Leave your answer in $\sum$ notation, do not evaluate.

Sol. to 1: By **independence** the number of sick days in a **three-year** period is also a **Poisson** random variable, but with mean $\mu = 10 + 10 + 10 = 30$. We need $P\{X \geq 36\}$.

Recall that $P\{X = k\} = e^{-\mu}\frac{\mu^k}{k!}$.

So the desired probability is

$$P\{X \geq 36\} = e^{-30} \sum_{k=36}^{\infty} \frac{30^k}{k!} \quad .$$

This is the **ans.**

Note: Maple says that it equals $0.1573834744\ldots$, but I did not expect you to calculate the exact value.

2. An actuary discovers that policyholders are four times as likely to file one claim as to file two claims. If the number of claims filed has a Poisson distribution, what is the standard deviation of the number of claims filed?

Sol. to 2: Let μ be the parameter of the Poisson distribution. We know that

$$P\{X = 1\} = e^{-\mu}\mu \quad , \quad P\{X = 2\} = e^{-\mu}\frac{\mu^2}{2!} = e^{-\mu}\mu^2/2 \quad .$$

The problem tell us that $P\{X = 1\}/P\{X = 2\} = 4$, hence

$$\frac{e^{-\mu}\mu}{e^{-\mu}\mu^2/2} = 4 \quad .$$

Simplifying the left ($e^{-\mu}$ cancels out), we get

$$\frac{2}{\mu} = 4 \quad .$$

Hence

$$\mu = \frac{2}{4} = \frac{1}{2} \quad .$$

Hence the parameter of the Poisson distribution is $\frac{1}{2}$. This means that both its expectation and variance equal $\frac{1}{2}$. But watch out! we need the **standard deviation**, that is the square-root of the variance, hence:

Ans. to 2: The standard deviation of the number of claims filed is $\sqrt{2}/2$.