Solutions to Math 477 "QUIZ" for Lecture 7

Version of Oct. 15, 2017 (Thanks to Rowen Kanj)

1 . The probability mass function of the discrete random variable X is

$$P\{X=0\}=0.1 \quad , \quad P\{X=1\}=0.5 \quad , \quad P\{X=2\}=0.4 \quad ,$$

and $P\{X=x\}=0$ if $x \notin \{0,1,2\}$. Find

(i) $E[2X^3-3X+1]$

(ii) $E[\cos(\pi X/2)]$

Sol. to 1(i):

$$E[2X^3-3X+1]=P\{X=0\}\cdot(2\cdot 0^3-3\cdot 0+1)+P\{X=1\}\cdot(2\cdot 1^3-3\cdot 1+1)+P\{X=2\}\cdot(2\cdot 2^3-3\cdot 2+1)$$

$$= 0.1\cdot(2\cdot 0^3-3\cdot 0+1)+0.5\cdot(2\cdot 1^3-3\cdot 1+1)+0.4\cdot(2\cdot 2^3-3\cdot 2+1)$$

$$= 0.1\cdot(1)+0.5\cdot(0)+0.4\cdot(11) = 0.1+4.4=4.5 \quad .$$

Ans to 1(i): 4.5.

Sol. to 1(ii):

$$E[\cos(\pi X/2)]=P\{X=0\}\cdot\cos(\pi\cdot 0/2)+P\{X=1\}\cdot\cos(\pi\cdot 1/2)+P\{X=2\}\cdot\cos(\pi\cdot 2/2)$$

$$0.1\cdot\cos(0)+0.5\cdot\cos(\pi/2)+0.4\cdot\cos(\pi)$$

$$= 0.1\cdot(1)+0.5\cdot(0)+0.4\cdot(-1) = -0.3 \quad .$$

Ans to 1(ii): -0.3.

2. Let X be the winnings of a gambler and assume that

$$P\{X=0\}=0.4 \quad ; \quad P\{X=1\}=0.2 \quad ; \quad P\{X=-1\}=0.3 \quad ; \quad P\{X=-2\}=0.1 \quad .$$

Find the variance $Var(X)$. Also find the standard deviation.

Sol. to 2: Since $Var(X)=E[X^2]-E[X]^2$, we need both $E[X]$ and $E[X^2]$.

$$E[X]=P\{X=0\}\cdot 0+P\{X=1\}\cdot 1+P\{X=-1\}\cdot(-1)+P\{X=-2\}\cdot(-2)$$

$$0.4\cdot 0+0.2\cdot 1+0.3\cdot(-1)+0.1\cdot(-2)=0+0.2-0.3-0.2=-0.3 \quad .$$

Next.

$$E[X^2] = P\{X=0\} \cdot 0^2 + P\{X=1\} \cdot 1^2 + P\{X=-1\} \cdot (-1)^2 + P\{X=-2\} \cdot (-2)^2$$

$$0.4 \cdot 0 + 0.2 \cdot 1 + 0.3 \cdot 1 + 0.1 \cdot 4 = 0 + 0.2 + 0.3 + 0.4 = 0.9 \quad .$$

Hence

$$Var(X) = E[X^2] - E[X]^2 = 0.9 - (-0.3)^2 = 0.9 - 0.09 = 0.81 \quad .$$

Finally the **standard deviation** is $\sqrt{Var(X)} = \sqrt{0.81} = 0.9$.

Ans. to 2: the variance, $Var(X)$ equals 0.81 and the standard deviation, σ, equals 0.9.