Solutions to Math 477 "QUIZ" for Lecture 6

Version of Sept. 27, 2017: Thanks to Judah Axelrod (who won a dollar) [correcting the error in the last line of 1(iii), and hence changing the final answer of 1(iii) from the erroneous one to the correct one]

1. The probability mass function of random variable X is given by $c/3^i$, $i = 0, 1, 2, \ldots$, where c is some positive value. Find (i) $P\{X = 0\}$, (ii)$P\{X < 3\}$, (iii) $P\{X > 5\}$.

Sol. of 1: First we must find c. Since *all* the probabilities must add-up to one, we must have

$$\sum_{i=0}^{\infty} P\{X = i\} = 1 \quad .$$

On the other hand, we are told that $P\{X = i\} = \frac{c}{3^i}$. Hence

$$\sum_{i=0}^{\infty} \frac{c}{3^i} = 1 \quad .$$

Sticking c out, we get,

$$c \sum_{i=0}^{\infty} \frac{1}{3^i} = 1 \quad .$$

Recall the **famous** (and very important!) fact from calc1,

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x} \qquad |x| < 1 \quad .$$

Here $x = \frac{1}{3}$, so

$$\sum_{i=0}^{\infty} \frac{1}{3^i} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2} \quad .$$

so $c\frac{3}{2} = 1$ implying that $c = \frac{2}{3}$. We now know that

$$P\{X = i\} = \frac{2}{3} \cdot (\frac{1}{3})^i \quad .$$

Now we know the exact formula for $P\{X = i\}$ we can answer the questions.

(i)

$$P\{X = 0\} = \frac{2}{3}(\frac{1}{3})^0 = \frac{2}{3} \quad .$$

(ii)

$$P\{X < 3\} = P(X = 0) + P(X = 1) + P(X = 2) = \frac{2}{3}(\frac{1}{3})^0 + \frac{2}{3}(\frac{1}{3})^1 + \frac{2}{3}(\frac{1}{3})^2$$

$$= \frac{2}{3}\left((\frac{1}{3})^0 + (\frac{1}{3})^1 + (\frac{1}{3})^2\right) = \frac{2}{3}\left(1 + \frac{1}{3} + \frac{1}{9}\right) = \frac{2}{3}\cdot\frac{9+3+1}{9} = \frac{26}{27} \quad .$$

(iii)

$$P\{X > 5\} = \frac{2}{3}\sum_{i=6}^{\infty}\frac{1}{3^i} = \frac{2}{3}\cdot\left(\frac{1}{3^6} + \frac{1}{3^7} + \frac{1}{3^8} + \dots\right)$$

$$= \frac{2}{3}\cdot\frac{1}{3^6}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\right) = \frac{2}{3}\cdot\frac{1}{3^6}\frac{1}{1-\frac{1}{3}} = \frac{2}{3}\cdot\frac{1}{3^6}\cdot\frac{3}{2} = \frac{1}{3^6} = \frac{1}{729} \quad .$$

Ans. to 1: (i) $\frac{2}{3}$, (ii) $\frac{26}{27}$, (iii) $\frac{1}{729}$.

2. Let X be the winnings of a gambler and assume that

$$P\{X = -2\} = 0.15 \quad , \quad P\{X = -1\} = 0.3 \quad , \quad P\{X = 1\} = 0.35 \quad , \quad P\{X = 2\} = 0.2 \quad ,$$

(a) Compute the conditional probability that gambler wins i, for $i = 1, 2$, given that he wins a positive amount.

(b) Find $E[X]$, his expected winning.

Sol. to 2(a): First let's compute $P\{X > 0\}$.

$$P\{X > 0\} = P\{X = 1\} + P\{X = 2\} = 0.35 + 0.2 = 0.55 \quad .$$

Hence

$$P(X = 1|X > 0) = \frac{P\{X = 1\}}{P\{X > 0\}} = \frac{0.35}{0.55} = \frac{7}{11} \quad , \quad P(X = 2|X > 0) = \frac{P\{X = 2\}}{P\{X > 0\}} = \frac{0.2}{0.55} = \frac{4}{11} \quad .$$

Sol. to 2(b):

$$E[X] = (-2)\cdot P\{X = -2\} + (-1)\cdot P\{X = -1\} + (1)\cdot P\{X = 1\} + (2)\cdot P\{X = 2\}$$

$$= (-2)\cdot 0.15 + (-1)\cdot 0.3 + (1)\cdot 0.35 + (2)\cdot 0.2 = 0.15 \quad .$$

Ans. to 2(b): The expected winning of the gambler, $E[X]$, equals 0.15.