Solutions to Math 477 "QUIZ" for Lecture 5

1. A loaded coin, whose probability of Heads is 0.6 is tossed 20 times. Assuming that the tosses are independent, what is the probability that

(i) it landed Heads all the time ?

(ii) it landed Heads 15 times and Tails 5 times?

Ans. to 1 (i) 0.6^{20} (ii) $\binom{20}{15}(0.6)^{15}(0.4)^5$.

Problem 2. Two soccer teams, A, and B compete. The probability that team A scores a goal is $\frac{3}{4}$. The team who scores any given goal is independent of who scored any other goal.

Calculate the probability that team B scored 3 goals before team A's 2^{nd} goal.

Sol. to 2. This is an instance of the **problem of the points**.

If Independent trials resulting in success with probability p and failure with probability $1-p$ are performed, then **the probability that** n **successes occur before** m **failures** is

$$\sum_{k=n}^{m+n-1} \binom{m+n-1}{k} p^k (1-p)^{m+n-1-k} \quad .$$

From the perspective of team B, $p = \frac{1}{4}$, $n = 3$ and $m = 2$, we have

$$\sum_{k=3}^{2+3-1} \binom{2+3-1}{k} (\frac{1}{4})^k (\frac{3}{4})^{2+3-1-k}$$

$$= \sum_{k=3}^{4} \binom{4}{k} (\frac{1}{4})^k (\frac{3}{4})^{4-k}$$

$$= \binom{4}{3} (\frac{1}{4})^3 (\frac{3}{4})^{4-3} + \binom{4}{4} (\frac{1}{4})^4 (\frac{3}{4})^{4-4}$$

$$4 \cdot (\frac{1}{4})^3 \frac{3}{4} + 1 \cdot (\frac{1}{4})^4 = \frac{13}{256} \quad .$$

Ans. to 2: The probability that team B scored 3 goals before team A's 2^{nd} goal is $\frac{13}{256}$.