Solutions to Math 477 "QUIZ" for Lecture 4

1. Let's assume that any child in any family is equally likely to be a boy or a girl.

In a family of four, what is the conditional probability that all the children are girls if

(i) the two older children are girls ;

(ii) the oldest and the youngest are girls ;

(iii) The family has at least one girl ;

(iv) The family has at least two girls .

Sol. to 1: We can use $P(A|B) = P(A \cap B)/P(B)$ but it is easier to look at the appropriate sample space at each problem.The event set has only **one** element, "all girls", i.e. the set E is always $\{GGGG\}$.

(i): The sample space S is $\{GGBB, GGBG, GGGB, GGGG\}$, hence the probability is $|E|/|S| = \frac{1}{4}$.

(ii): The sample space S is $\{GBBG, GBGG, GGBG, GGGG\}$, hence the probability is $|E|/|S| = \frac{1}{4}$.

(iii): The sample space S is $\{BBBB\}^c$, hence the probability is $|E|/|S| = \frac{1}{16-1} = \frac{1}{15}$.

(iv): The sample space S is $\{BBBB, BGGG, GBGG, GGBG, GGGB\}^c$, hence the probability is $|E|/|S| = \frac{1}{16-5} = \frac{1}{11}$.

2. A house is shared by ten people, four of them are women and six of them are men. They are each equally likely to visit the bathroom. Let's assume that no one changes the position of the toilet seat after doing their business.

If a woman goes to the bathroom, the toilet seat is always down, but if a man goes to the bathroom, with probability 0.3 the toilet seat is down, and with probability 0.7 the toilet seat is up.

You enter the bathroom and you find that the toilet seat is down. What is the probability that the previous person was a man?

Sol. of 2: Let M be the event "it is a man", and W be the event that "it is a woman". Let D be the event "toilet seat down", and let U be the event "toilet seat up".

The data of the problem is:

$$P(M) = \frac{6}{10} = 0.6 \quad , \quad P(W) = \frac{4}{10} = 0.4 \quad ;$$

$$P(D|W) = 1 \quad , \quad P(U|W) = 0 \quad ;$$

$$P(D|M) = 0.3 \quad , \quad P(U|M) = 0.7 \quad .$$

Hence

$$P(D) = P(D|M) \cdot P(M) + P(D|W) \cdot P(W) = 0.3 \cdot 0.6 + 1 \cdot 0.4 = 0.58 \quad .$$

By **Bayes' Formula**:

$$P(M|D) = \frac{P(D|M) \cdot P(M)}{P(D|M) \cdot P(M) + P(D|W) \cdot P(W)} = \frac{0.3 \cdot 0.6}{0.58} = \frac{0.18}{0.58} = \frac{9}{29} = 0.3103448276\ldots$$

Ans. to 2.: If you enter the bathroom in that house and notice that the toilet seat is down, the probability that the previous person was a man is $\frac{9}{29}$, about %31.034 .