Solutions to Math 477(3) "QUIZ" for Lecture 3

1. What is the probability that if you roll a fair die 10 times, you will get neither 2s nor 3s?

Sol. of 1: The **sample space**, S, is $\{1,2,3,4,5,6\}^{10}$ and its number of elements, by the **multiplication principle**, is:

$$|S| = |\{1,2,3,4,5,6\}^{10}| = |\{1,2,3,4,5,6\}|^{10} = 6^{10} \quad .$$

We are interested in the set of rolls where **only** the members of $\{1,4,5,6\}$ are allowed to show up, let's call this set E. E is the set $\{1,4,5,6\}^n$. Also by the multiplication principle:

$$|E| = |\{1,4,5,6\}^{10}| = |\{1,4,5,6\}|^{10} = 4^{10} \quad .$$

Since we have a **fair** die, all outcomes are **equally likely**, hence the desired probability is

$$\frac{|E|}{|S|} = \frac{4^{10}}{6^{10}} = (\frac{4}{6})^{10} = (\frac{2}{3})^{10} \quad .$$

Ans. to 1: The probability that if you roll a fair die 10 times, you will get neither 2s nor 3s is $(\frac{2}{3})^{10}$.

Note: In this section we did this problem via **combinatorics**. Later, in this class, we will see a way to solve this problem using the notion of **independent events**.

2. If you have a class with 20 Freshmen, 30 Sophmores, and 40 Juniors, and you pick 10 random students to give them A-s. What is the probability that exactly 5 Freshmem, 2 Sophmores , and 3 Juniors will get an A. Assume that everyone is equally likely to be picked.

Sol. to 2: The total number of students in the class is $20+30+40=90$. The **sample space**, S is the set of 10-element subsets, so

$$|S| = \binom{90}{10} \quad .$$

The set that we are intersted in is the **Cartesian product** of three sets

- 5-element subsets of a set of 20 freshmen (whose number is $\binom{20}{5}$)
- 2-element subsets of a set of 30 sophmores (whose number is $\binom{30}{2}$)
- 3-element subsets of a set of 40 sophmores (whose number is $\binom{40}{3}$)

By the **multiplication principle**, the size of the desired set, let's call it E, is the product, so

$$|E| = \binom{20}{5}\binom{30}{2}\binom{40}{3} \quad .$$

Since every choice is assumed **equally likely**, the desired probability is

$$P(E) = \frac{|E|}{|S|} = \frac{\binom{20}{5}\binom{30}{2}\binom{40}{3}}{\binom{90}{10}} \quad .$$

Ans. to 2: The probability that exactly 5 Freshmem, 2 Sophmores , and 3 Juniors will get an A is $\frac{\binom{20}{5}\binom{30}{2}\binom{40}{3}}{\binom{90}{10}}$.

Note: In you want to find the actual number, go to Maple, and type

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binomial(20,5)*binomial(30,2)*binomial(40,3)/binomial(90,10);
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and you would get $\frac{15017600}{1289304819}$. To get the answer in decimals, you do

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evalf(binomial(20,5)*binomial(30,2)*binomial(40,3)/binomial(90,10));
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and get 0.01164782740, so the probability is about %1.16478 .

(Of course, in a quiz or test, I don't expect you to evaluate it.)

Comment:I did not have a chance to do this type of problem in class, but a similar problem is in the handout L3.pdf . About one half of the students got it right, but some did not attempt it, and some did gibberish. In particular some people added and wrote on the top $\binom{20}{5} + \binom{30}{2} + \binom{40}{3}$. **Do not confuse addition and multiplication**. In this problem it is a Cartesian product!, so you **multiply**.