Solution to Dr. Z.'s Math 477 "QUIZ" for Lecture 2

1. Out of a class of 220 students,

• 100 students play football

• 100 students play soccer

• 100 students play basketball

• 40 students play football and soccer

• 40 students play football and basketball

• 40 students play soccer and basketball

• 30 students play football soccer and basketball .

How many students play none of the three sports?

Sol. to 1: Let F, S, and B , respectively, be the sets of students who play football, soccer, and basketball. We need to find the number of elements in the set $F^c S^c B^c$. By **PIE** (U is the universal set of all students)

$$|F^c S^c B^c| = |U| - (|F| + |S| + |B|) + (|FS| + |FB| + |SB|) - |FSB|$$

$$= 220 - (100 + 100 + 100) + (40 + 40 + 40) - 30 = 220 - 300 + 120 - 30 = 10 \quad .$$

Ans. to 1: ten students play none of the three sports.

Commments: Many people got it right, but some people did the probability version getting the correct answer that the probability is $\frac{1}{22}$, but **not** answering the question. You should answer the **question** that was asked. Some people used the probability version, got $\frac{1}{22}$ and then multiplied it by 220 getting that the final answer is 10. This is perfectly legit, although not as elegant as using the countiong version of PIE.

2. Out of a large group of students it is found that 20% take both calculus and algebra, and 30% take neither of them. The probability that a student takes calculus is twice the probability that a student takes algebra.

Determine the probability that a randomly chosen member of this group takes algebra?

Sol. to 2: Let C be the set of students who take calculus and A be the set of students who take algebra.

We have to find $P(A)$, let's call is x. By the date of the problem

$$P(AC) = 0.2 \quad , \quad P(A^c C^c) = 0.3 \quad .$$

Also, by definition of x, $P(A) = x$, and since $P(C) = 2P(A) = 2x$.

By **PIE** for two sets
$$P(A^c C^c) = 1 - P(A) - P(C) + P(AC) \quad .$$

Plugging the above, we get the equation
$$0.3 = 1 - x - 2x + 0.2 \quad .$$

Simplifying
$$3x = 0.9 \quad ,$$

dividing by 3, we get that $x = 0.3$. Hence $P(A) = 0.3$.

Ans. to 2.: The probability that a randomly chosen member of this group takes algebra is %30.

Note: Most people got it right, but by sheer coincidence, I solved this problem in class (with French and German instead of algebra and calclulus).