Solutions to Math 477 "QUIZ" for Lecture 23

1. If 1000 fair dice are rolled, find the approximate probability that the average number of dots is between 3.5 and 3.55.

Sol. of 1: For a **single** throw of one die, $\mu = \frac{7}{2}$ and

$$\sigma^2 = \frac{1}{6}\left(\sum_{i=1}^{6} i^2\right) - \mu^2$$

$$= \frac{1}{6}\frac{6 \cdot 7 \cdot (2 \cdot 6 + 1)}{6} - (\frac{7}{2})^2 = \frac{91}{6} - (\frac{7}{2})^2 = \frac{35}{12} \quad .$$

Hence the average of 1000 throws has mean $\frac{7}{2}$ and $\sigma = \frac{\sqrt{35/12}}{\sqrt{1000}} = 0.05400617252\ldots$

This is approximated by a Normal r.v with $\mu = 3.5$ and $\sigma = 0.05400617252\ldots$

Hence the desired probability is approximately

$$P\{3.5 < X < 3.55\} = P\{0 < X - 3.5 < 3.55 - 3.5\} = P\{0 < X - 3.5 < 0.05\}$$

$$= P\{0 < \frac{X - 3.5}{\sigma} < \frac{0.05}{0.05400617252\ldots}\}$$

$$= P(0 < Z < 0.9258200990) = \Phi(0.9258200990) - \Phi(0) = 0.8227302599 - 0.5 = 0.3227302599\ldots \quad .$$

Ans. to 1: The probability that the average number of dots is between 3.5 and 3.55 is 0.3227302599...
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2. State the Strong Law of Large Numbers.

Sol. to 2: Let $X_1, X_2, \ldots,$ be a sequence of independent and identically distributed (aka iid) random variables, each having finite mean μ. Then with probability 1,

$$\frac{X_1 + \ldots + X_n}{n} \to \mu \quad as \quad n \to \infty \quad .$$