Solutions to Math 477 "QUIZ" for Lecture 22

1. State and Prove Markov's inequality

Sol. of 1: If X is a random variable that takes only non-negative values, then for any $a > 0$:

$$P\{X \geq a\} \leq \frac{E[X]}{a} \quad .$$

Proof: For $a > 0$, let

$$I = \begin{cases} 1 & , \quad if \quad X \geq a\,; \\ 0 & , \quad otherwise. \end{cases} \quad .$$

Obviously (since $X \geq 0$)

$$I \leq \frac{X}{a} \quad .$$

(When $X < a$ the left side is 0 and the right side is non-negative, when $X \geq a$ the left side is 1 and the right side is ≥ 1 (and usually, > 1). Taking expectation, we have

$$E[I] \leq E[\frac{X}{a}] = \frac{E[X]}{a} \quad .$$

But $E[I] = 0 \cdot P[X < a] + 1 \cdot P[X \geq a] = P[X \geq a]$, so the inequality follows.

2. State Chebychev's inequality.

Sol. of 2: If X is random variable with mean μ and variance σ^2, then for any value $k > 0$,

$$P\{|X - \mu| \geq k\} \leq \frac{\sigma^2}{k^2} \quad .$$