

Solutions to Math 477 “QUIZ” for Lecture 21

Version of Dec. 17, 2017 (Thanks to Mitchell Bregman)

1. The value of a piece of factory equipment after two years of use is $10(0.2)^X$, where X is a random variable having moment generating function

$$M_X(t) = \frac{1}{1-3t} \quad , \quad \text{for } t < \frac{1}{3} \quad .$$

Calculate the expected value of this piece of equipment after two years of use.

Sol. to 1:

$$E[10(0.2)^X] = 10 E[(0.2)^X] = 10E[e^{(\ln 0.2)X}] = 10M_X(\ln 0.2) = 10 \cdot \frac{1}{1-3(\ln 0.2)} = 1.715762132 \dots \quad .$$

Ans. to 1: The expected value of this piece of equipment after two years of use is $1.715762132 \dots$.

2. X and Y are independent random variables with common moment generating function $M(t) = e^{t^4}$.

Let $W = 2X + Y$ and $Z = Y - X$.

Determine the joint moment generating function, $M(t_1, t_2)$ of W and Z .

Sol. to 2:

$$\begin{aligned} M_{W,Z}(t_1, t_2) &= E[e^{t_1 W + t_2 Z}] = E[e^{t_1 (2X+Y) + t_2 (Y-X)}] = E[e^{(2t_1-t_2)X + (t_1+t_2)Y}] = \\ &= M_X(2t_1 - t_2) M_Y(t_1 + t_2) = e^{(2t_1-t_2)^4} e^{(t_1+t_2)^4} = e^{(2t_1-t_2)^4 + (t_1+t_2)^4} \quad . \end{aligned}$$