1. In a certain family of three boys, the Height (in centimeters) and Weights (in kilograms) are as follows

Alex: Height: 170cm ; Weight: 75kg

Bob: Height: 180cm ; Weight: 80 kg

Charlie: Height: 190cm ; Weight: 85kg

What is The correlation between the Height and the Weight in that family?

Sol. to 1:

$$\mu_{Height} = E[Height] = \frac{1}{3}(170 + 180 + 190) = 180 \quad.$$

$$\mu_{Weight} = E[Weight] = \frac{1}{3}(75 + 80 + 85) = 80 \quad.$$

$$Var(Height) = \frac{1}{3} \cdot ((170 - 180)^2 + (180 - 180)^2 + (190 - 180)^2) = \frac{200}{3}$$

$$Var(Weight) = \frac{1}{3} \cdot ((75 - 80)^2 + (80 - 80)^2 + (85 - 80)^2) = \frac{50}{3}$$

Also

$$Cov(Height, Weight) = \frac{1}{3} \cdot ((170-180)\cdot(75-80)+(180-180)\cdot(80-80)+(190-180)\cdot(85-80)) = \frac{100}{3}$$

Finally

$$Corr(Height, Weight) = \frac{Cov(Height, Weight)}{\sqrt{Var(Height)}\sqrt{Var(Weight)}} = \frac{100/3}{\sqrt{(200/3)\cdot(50/3)}} = 1 \quad.$$

Ans. to 1: The correlation between the Height and the Weight in the family is 1, in other words, it is perfect!

2. Suppose that $Var(X) = 2$, $Var(Y) = 1$, $Var(X + Y) = 4$. Find $Var(2X + 3Y)$.

Sol. to 2: We first use the important formula

$$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y) \quad,$$

in order to compute $Cov(X, Y)$. We have

$$4 = 2 + 1 + 2Cov(X, Y) \quad,$$

giving that

$$Cov(X, Y) = \frac{1}{2} \quad.$$

Next:

$$Var(2X+3Y) \; = \; Var(2X) + Var(3Y) + 2Cov(2X, 3Y) \; = \; 2^2 \cdot Var(X) + 3^2 \cdot Var(Y) + 2 \cdot 2 \cdot 3 \cdot Cov(X, Y) \; =$$

$$4 \cdot Var(X) \; + \; 9 \cdot Var(Y) + 12 \cdot Cov(X, Y) \; = \; 4 \cdot 2 \; + \; 9 \cdot 1 + 12 \cdot \frac{1}{2} \; = \; 23 \quad .$$

Ans. to 2: $Var(2\,X + 3\,Y) = 23$.