Solution to Dr. Z.'s Math 477 "QUIZ" for Lecture 1

1. A class has 10 boys and 6 girls. You have to pick a committee of 5 boys and 3 girls. In how many ways can you do it?

Sol. to 1.: We have to make two **independent** decisions.

- The number of ways of choosing 5 boys out 10 is $\binom{10}{5}$

- The number of ways of choosing 3 girls out 6 is $\binom{6}{3}$.

The total number of such committees is the number of elements in their **Cartesian product**, i.e. obtained by multiplying them. Hence it is

Ans. to 1.: $\binom{10}{5}\binom{6}{3}$. **Note**: If you want the actual number, we have

$$\binom{10}{5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5!} = 252 \quad .$$

$$\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3!} = 20 \quad .$$

This evaluates to $252 \cdot 20 = 5040$ (but you would get full credit without evaluating it, unless I *specifically* ask for it).

2. In how many ways can you roll a pair of dice so that the sum of the dots is neither 3 nor 5.

Sol. of 2. It is easier to find the number of pairs that sum up to 3 and the number of pairs that sum up to 5 (obviously there is no overlap, you can't add-up to 3 and add-up to 5).

Adding up to 3: $\{[1,2],[2,1]\}$.

Adding up to 5: $\{[1,4],[2,3],[3,2],[4,1]\}$.

So the number of pairs that ad up to 3 or add-up to 5 is $2+4=6$. Hene the number of pairs that neither add-up to 3 nor add-up to 5 is $6^2-6=36-6=30$.

Ans. to 2.: The number of ways that you can roll a pair of dice so that the sum of the dots is neither 3 nor 5 is 30.